Misread mysteries of the Quantum*

U. A. Yajnik
Physics Department, IIT, Bombay

April 2002

1 An *un*-natural philosophy?

Quantum Mechanics created a major revolution in our understanding of the physical world. Newtonian world was the world of daily experience. Newtonian formulation made this understanding mathematical. This was termed Natural Philosophy. But Quantum Mechanics was at first a completely different world. Its rules had to be intuited indirectly. Mathematical formulation was even a greater exercise.

It is no surprise then that even the stalwart pioneers of the subject remained somewhat unsettled about what exactly it all meant. Philosophers put it across by distinguishing between Ontology and Epistemology. Ontology refers to the stuff that is out there. Epistemology is how our knowledge is organised. In this sense the Newtonian advance was epistemic in nature. Ontology of what he talked about was self evident or easily verifiable. Quantum Mechanics however so puzzled its very proponents that they remained unsure what ontology there was. Little intuition could be gained over and above what the rules of calculation allowed. Was one to identify ontology with epistemy when one reached the microscopic world?

2 Riding the waves of Uncertainty

The phenomena presented by the Quantum world were so unfamiliar that it took almost two generations to digest them. Finally when it first emerged, the understanding seemed to suggest some kind of wave phenomenon. Electron diffraction could be understood as arising from free electron waves and spectral lines from standing waves. The famous ψ was invented. But its interpretation was ticklish. It was a *probability* amplitude wave. But we thought we were doing mechanics and not tossing dice. Unfortunately the wave picture persisted for very long. And probabilities continued to bother the great masters of the subject.

*contribution to the Physics Association magazine *Reflections*

The real essence of the new science was grasped by Dirac right at the inception. In his classic textbook written in 1929 Dirac emphasised the abstract approach. He begins by observing that there should be no surprise that a new constant of nature h presents itself; otherwise there would be no real distinction between the micro and the macro worlds. No fundamental scales would distinguish the two, and in principle the same stories we know at large scale should boringly repeat however finely we probed.

He then goes on to emphasise the new principle Quantum Mechanics presents to us. This is the Principle of Superposition of States. Classically we cannot imagine superposing two potential states of a system. Either the ball is inside the boundary or outside. It does not exist in a superposition of these states. States in QM are, like in Classical Mechanics, labelled by the possible values of observables. But with the difference that some of the classically allowed states may not occur and vice versa. We are to list all the possible observables of a system and their possible values, these are called eigenstates. And the general state of the system would be a superposition of the various eigenstates. The mathematical structure is therefore of a linear vector space.

The persistence of the wave picture had to do with this Principle. But this is the new principle revealed by QM. There is nothing uncertain about it. Nor is there any uncertainty in how the states evolve. Time evolution of the state is completely deterministic, by a first order differential equation. Identically prepared systems could however result in different answers in individual experiments because the system when observed can only be found in one its eigenstates. This is the result of a radically distinct way that states are listed in QM. An uneasy feeling of uncertainty results from this very concrete and crisp principle. The problem clearly lies in the eyes of the beholder.

There is no such thing as “wave”function “collapse.” By the latter is usually meant how the system is “forced” into one of its eigenstates upon “observation”. By “observation” is meant a macroscopic apparatus that interacts with a particular observable associated with the quantal system. Actually we may turn the definition around. Whatever interaction always leaves behind the system in a specific eigenstate is an act of observation. But there are other interactions allowed which do not “collapse” the “wave”function. They alter how the state is made of its eigenstates without reducing it to a pure eigenstate. But you say, anything macroscopic always leads to collapse so how can we think of it in the same class as a quantal interaction? Well, there are macroscopic entities that do not collapse the wavefunction. One of them is Gravity. This is a classical field for all practical purposes. It can never be switched off, i.e., no system can be isolated from it. If Gravity were an observer it would have reduced the entire Universe to a single eigenstate by now. We can also easily construct classical, macroscopic arrangements of electromagnetic fields which will not collapse a wavefunction.

3 Identity crisis : a new principle

There is indeed yet another principle, about the peculiar listing of states, which occurs for an assembly of quanta. This principle was first proposed by S. N. Bose as the the fundamental indistinguishability of photons. The uncanniness of this principle is best brought out by considering a simple heads-or-tails experiment with two Quantum coins. Classical indistinguishability will imply that the HH and the TT possibilities have weightage 1/4 each, while the HT (indistinguishable from TH) has the weightage 1/2 because it can be arrived at in two different ways. But this is where quanta differ. They are so strictly indistinguishable that there is no point even conceiving of two different configurations. Thus two bosonic coins will have the three possibilities, each with weightage exactly 1/3. It gets even more mysterious with fermions. Pauli exclusion principle will insist that there is only one possible state for the two coin system, viz., HT, being completely equivalent to what we may have called TH, and it has weightage exactly 1. No other states are possible.

We see that this "identity crisis" – absence of any "individual" identity of quanta – should actually be understood as a property of the space of eigenstates available to a set of quanta. We should avoid first introducing distinct particles and then insisting they are indistinguishable. Quanta are therefore not "particles" at all. "Number of quanta" is however an approximate observable of many systems. Or more correctly put, only those systems have a classical analogue of being a collection of "particles" that permit an approximately conserved number operator. Indeed photons are the most familiar quanta that have no conserved number. Their states are superpositions of states with different values of the number. Only under special conditions do we observe states with a few or a definite number of photons.

Fermions obey the Exclusion Principle. As a result far fewer number of states are available to them than to similar number of classical particles. In most practical applications this is expressed as the presence of "exchange coupling" or "degeneracy pressure". But experts know that there is no force involved. This is simply the kinematics of fermions, not their dynamics.

In summary it is not Uncertainty or Wave-Particle Duality that are so important. It is the Superposition Principle and the different listing of states. Furthermore, exactly identical quanta is a feature of the microscopic world. Quantum Mechanics is not seen in its full glory until it is stated for assemblies of identical quanta.

4 Uncharted domains

One intriguing feature of Quantal systems however seems to be the ease with which we can "quantize" them. After all the mystery, we have a nice recipe - replace Poisson Brackets by commutators, and extend the expressions for classical observables judiciously to a quantum operators. This really is magic. But we soon realise that the situation is quite the reverse. We have been able to tackle only those systems that admit a classical limit that permits a canonical structure. There could easily be quantal systems that do not have such a limit and may have hitherto been unobserved. After all, world was Quantum first, it is our limitation that only classical limits are observable.

One example in fact stares us in the face. One learns in advanced Quantum Mechanics that fermions have to be "quantized" by *anti*-commutators, i.e., with rules involving AB+BA rather than AB-BA. This has simple connection to the Exclusion Principle. Further the field operators one introduces to represent a collection of fermions have no classical limit. Only quadratic expressions in these operators have classical limits, for example electric current for electrons is a quadratic expression in the underlying field operators. In dealing with the mathematics of such operators one does introduce classical numbers, but then they are Grassmann or anti-commuting "numbers", not accessible on the number line or the complex plane.

There are other bizarre situations thrown up by Quantum Mechanics. The quarks that constitute the neutrons and protons cannot *in principle* be observed isolated. This is because their self-interaction is so strong that it modified the ground state of the system. Unlike a potential well which particles might roll into, here it is like a phase transition. The phase with a pair of free quarks is infinitely higher in energy than the one where the pair is bound up in a slew of gluons and quarks.

Perhaps we are yet to uncover a large number of situations in which the newly discovered principles, only a century old, can be manifested.

5 Life without fermions?

We have tried to bring out the fact that the oddities of Quantum Mechanics emphasised in M.Sc. education are really misleading. The true oddities are rather well defined principles, no less bizarre in their own right. Further, unless the principles of many-particle QM are brought out, the principles are only partially stated. One might think we are overstating the obvious. A caveat we make is, try to imagine a "wave mechanical" world but in which there was no Exclusion Principle. Before you launch into this recall the valence and the band structure! And don't forget ferromagnets! Best luck!!