

# **SLUO LECTURE SERIES**

## ***Electronics Basic Concepts II***

### **LECTURE # 11**

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*Lectures on Detector Techniques*  
*Stanford Linear Accelerator Center*  
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**Electronics II**  
**Timing Measurements and Signal Digitization**

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*copies of transparencies in pdf format at*  
*<http://www-physics.lbl.gov/~spieler>*

*for more details see UC Berkeley Physics 198 course notes*  
*at [http://www-physics.lbl.gov/~spieler/physics\\_198\\_notes](http://www-physics.lbl.gov/~spieler/physics_198_notes)*

## Summary of Previous Discussions

### 1. Equivalent Noise Charge is a derived quantity

Although the primary energy deposition corresponds to a fixed charge, the detected signal is an induced current.

### 2. The obtainable signal-to-noise ratio is the same for voltage-, current-, or charge-sensing amplifiers.

### 3. Electronic noise is determined by two types of noise sources

- equivalent noise voltage
- equivalent noise current

that together with the

- total capacitance at the input node and the
- pulse shaper

determine the equivalent noise charge

$$Q_n^2 = i_n^2 T_s F_i + C_i^2 v_n^2 \frac{F_v}{T_s} + C_i^2 F_f A_f$$

- The first term – the noise current – increases with shaping time.
- The second term – the voltage noise - increases with capacitance and decreases with shaping time.
- The third term – 1/f voltage noise – increases with capacitance (as it is a voltage noise source), but is independent of shaping time.

The shaper noise indices (“shape factors”)  $F_i$ ,  $F_v$ ,  $F_f$  can be calculated in either the frequency or time domain. Most time variant shapers can only be evaluated in the time domain.

### 4. In all systems the optimum noise charge is obtained when the current and voltage noise contributions are equal.

BJTs: optimum operating current (distinct minimum)  
 minimum noise independent of shaping time,

whereas for devices where the noise current is negligible,

FETs: noise decreases with increasing operating current and  
 shaping time

5. In general, for a given combination of noise current and voltage, the noise improves with decreasing capacitance.

In special cases – where the input noise voltage and the device capacitance are correlated – the minimum noise is obtained when the scalable device capacitance matches the fixed input load capacitance (detector + strays).

### **Capacitive matching is not a universal optimization criterion!**

In optimized systems

$$Q_{n,opt} \propto \frac{1}{\sqrt{C_{det}}}$$

6. In systems optimized for low power the power required in the input transistor for a given signal-to-noise ratio scales with the square of the total input capacitance

$$P \propto C_{tot}^2$$

⇒ feasibility of highly segmented detectors (pixel arrays)

7. Noise properties of transistors are subject to radiation effects

BJTs: degradation of current gain  
 (increased base current ⇒ shot noise)  
 can be mitigated by reducing operating current

FETs: increase in noise coefficient  $\gamma_n$ .  
 PMOS devices show less degradation

## Timing Measurements

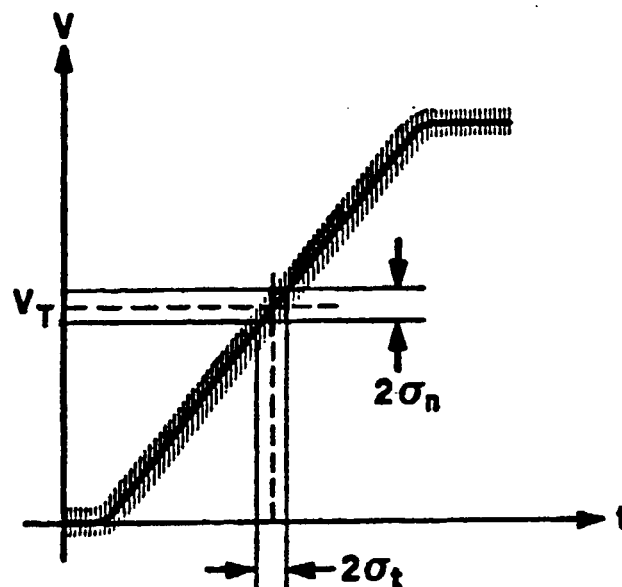
Pulse height measurements discussed up to now emphasize accurate measurement of signal charge.

- Timing measurements optimize determination of time of occurrence.
- For timing, the figure of merit is not signal-to-noise, but slope-to-noise ratio.

Consider the leading edge of a pulse fed into a threshold discriminator (comparator).

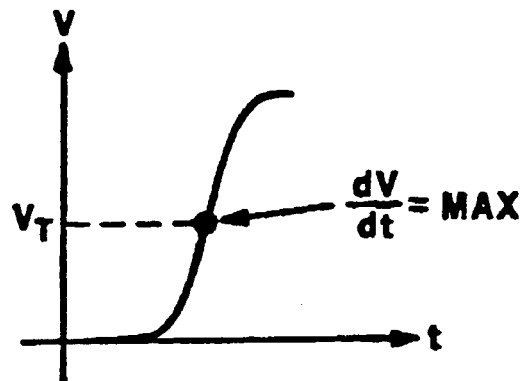
The instantaneous signal level is modulated by noise.

⇒ time of threshold crossing fluctuates



$$\sigma_t = \frac{\sigma_n}{\left. \frac{dV}{dt} \right|_{V_T}}$$

Typically, the leading edge is not linear, so the optimum trigger level is the point of maximum slope.



## Pulse Shaping

Consider a system whose bandwidth is determined by a single  $RC$  integrator.

The time constant of the  $RC$  low-pass filter determines the

- rise time (and hence  $dV/dt$ )
- amplifier bandwidth (and hence the noise)

Time dependence

$$V_o(t) = V_0(1 - e^{-t/\tau})$$

The rise time is commonly expressed as the interval between the points of 10% and 90% amplitude

$$t_r = 2.2\tau$$

In terms of bandwidth

$$t_r = 2.2\tau = \frac{2.2}{2\pi f_u} = \frac{0.35}{f_u}$$

Example: An oscilloscope with 100 MHz bandwidth has 3.5 ns rise time.

The frequency response of this simple system is

$$A(i\omega) \equiv \frac{V_o}{V_i} = \frac{A_0}{1 + i\omega\tau}$$

The magnitude of the gain

$$A(\omega) = \frac{A_0}{\sqrt{1 + (\omega\tau)^2}}$$

At the upper bandwidth limit

$$\omega\tau = 1 \quad \rightarrow \quad f_u = \frac{1}{2\pi\tau}$$

the signal response has dropped to

$$A(f_u) = \frac{A_0}{\sqrt{2}}$$

Expressed in terms of the upper bandwidth limit  $f_u$ , the frequency response

$$A(f) = \frac{A_0}{\sqrt{1 + (f / f_u)^2}}$$

## A Parenthetical Practical Comment

Since it is often convenient to view the frequency response logarithmically, gain is often expressed in logarithmic scale, whose unit is the Bel.

Defined as a power ratio

$$A[\text{B}] = \log \frac{P_2}{P_1}$$

Since dB tends to be a more common order of magnitude, this is usually written

$$A[\text{dB}] = 10 \log \frac{P_2}{P_1}$$

which can be expressed as a voltage ratio

$$A[\text{dB}] = 10 \log \frac{V_2^2 / R_2}{V_1^2 / R_1} = 20 \log \frac{V_2}{V_1} + 10 \log \frac{R_1}{R_2}$$

or if  $R_1 = R_2$

$$A[\text{dB}] = 20 \log \frac{V_2}{V_1}$$

$V_2/V_1 = 10$  corresponds to 20 dB.

**Caution:** In practice, voltage gains are often expressed in dB without regard to the resistance ratio. Clearly, converting such a gain figure into a power ratio can be very misleading.

At the upper cutoff frequency of the amplifier, the gain has dropped to  $1/\sqrt{2}$  of its maximum, corresponding to  $-3$  dB.

⇒ Bandwidth limits are often referred to colloquially as “3 dB frequencies”.



## Bandwidth of a Cascade of Amplifiers

Invariably, the required gain is provided by multiple amplifying stages.

If we define the bandwidth of a cascade of  $n$  amplifiers  $f_u^{(n)}$  as the frequency where the gain has dropped by  $-3$  dB, i.e.  $1/\sqrt{2}$

$$\left[ \frac{1}{\sqrt{1 + (f_u^{(n)} / f_u)^2}} \right]^n = \frac{1}{\sqrt{2}}$$

then

$$\sqrt{1 + (f_u^{(n)} / f_u)^2} = \sqrt[n]{2}$$

$$\frac{f_u^{(n)}}{f_u} = \sqrt[n]{2^{1/n} - 1}$$

Correspondingly, for the lower cutoff frequency

$$\frac{f_l^{(n)}}{f} = \frac{1}{\sqrt[n]{2^{1/n} - 1}}$$

Calculating the rise time of a cascade of  $n$  stages is more difficult, but to a good approximation ( $\sim 10\%$ )

$$t_r \approx \sqrt{t_{r1}^2 + t_{r2}^2 + \dots + t_{rn}^2}$$

## Choice of Rise Time in a Timing System

Assume a detector pulse with peak amplitude  $V_0$  and a rise time  $t_c$  passing through an amplifier chain with a rise time  $t_{ra}$ .

The cumulative rise time at the discriminator input is

$$t_r = \sqrt{t_c^2 + t_{ra}^2}$$

The electronic noise at the amplifier output is

$$V_{no}^2 = \int v_{ni}^2 df = v_{ni}^2 \Delta f_n$$

For a single RC time constant the noise bandwidth

$$\Delta f_n = \frac{\pi}{2} f_u = \frac{1}{4\tau} = \frac{0.55}{t_{ra}}$$

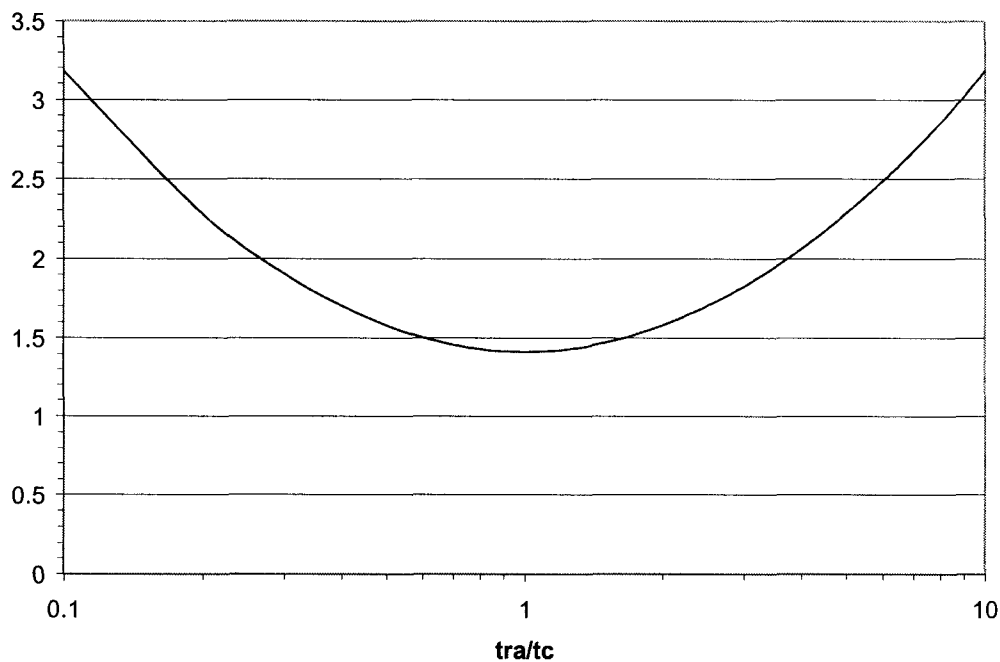
As the number of cascaded stages increases, the noise bandwidth approaches the signal bandwidth. In any case

$$\Delta f_n \propto \frac{1}{t_{ra}}$$

## The timing jitter

$$\sigma_t = \frac{V_{no}}{dV/dt} \approx \frac{V_{no}}{V_0/t_r} = \frac{1}{V_0} V_{no} t_r \propto \frac{1}{V_0} \frac{1}{\sqrt{t_{ra}}} \sqrt{t_c^2 + t_{ra}^2} = \frac{\sqrt{t_c}}{V_0} \sqrt{\frac{t_c}{t_{ra}} + \frac{t_{ra}}{t_c}}$$

The second factor assumes a minimum when the rise time of the amplifier equals the collection time of the detector  $t_{ra} = t_c$ .



At amplifier rise times greater than the collection time, the time resolution suffers because of rise time degradation. For smaller amplifier rise times the electronic noise dominates.

The timing resolution improves with decreasing collection time  $\sqrt{t_c}$  and increasing signal amplitude  $V_0$ .

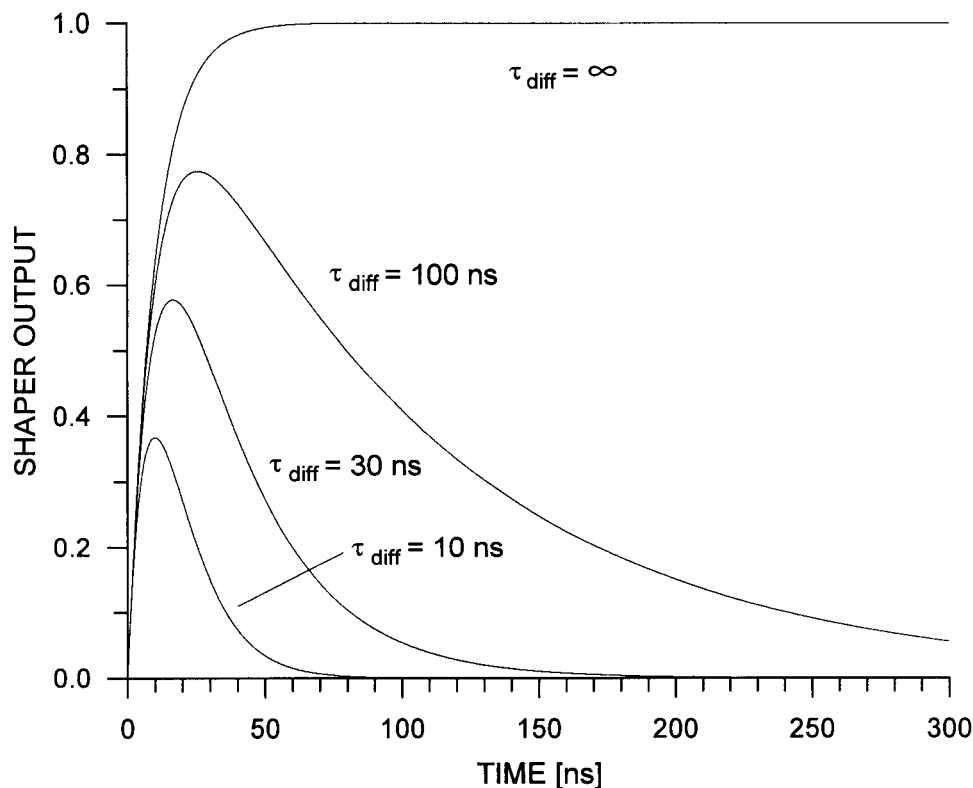
The integration time should be chosen to match the rise time.

How should the differentiation time be chosen?

As shown in the figure below, the loss in signal can be appreciable even for rather large ratios  $\tau_{diff}/\tau_{int}$ , e.g. >20% for  $\tau_{diff}/\tau_{int} = 10$ .

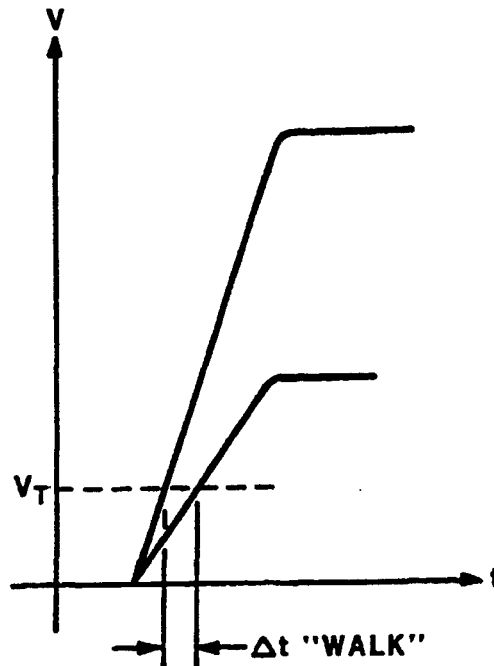
Since the time resolution improves directly with increasing peak signal amplitude, the differentiation time should be set to be as large as allowed by the required event rate.

**CR-RC SHAPER**  
**FIXED INTEGRATOR TIME CONSTANT = 10 ns**  
**DIFFERENTIATOR TIME CONSTANT =  $\infty$ , 100, 30 and 10 ns**



## Time Walk

For a fixed trigger level the time of threshold crossing depends on pulse amplitude.



⇒ Accuracy of timing measurement limited by

- jitter (due to noise)
- time walk (due to amplitude variations)

If the rise time is known, “time walk” can be compensated in software event-by-event by measuring the pulse height and correcting the time measurement.

This technique fails if both amplitude and rise time vary, as is common.

In hardware, time walk can be reduced by setting the threshold to the lowest practical level, or by using amplitude compensation circuitry, e.g. constant fraction triggering.

## Lowest Practical Threshold

Single  $RC$  integrator has maximum slope at  $t=0$ .

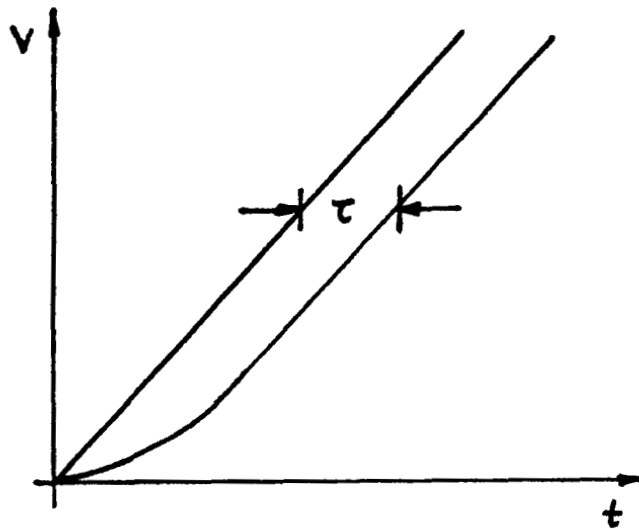
$$\frac{d}{dt}(1 - e^{-t/\tau}) = t e^{-t/\tau}$$

However, the rise time of practically all fast timing systems is determined by multiple time constants.

For small  $t$  the slope at the output of a single  $RC$  integrator is linear, so initially the pulse can be approximated by a ramp  $\alpha t$ .

Response of the following integrator

$$V_i = \alpha t \rightarrow V_o = \alpha(t - \tau) - \alpha \tau e^{-t/\tau}$$



⇒ The output is delayed by  $\tau$  and curvature is introduced at small  $t$ .

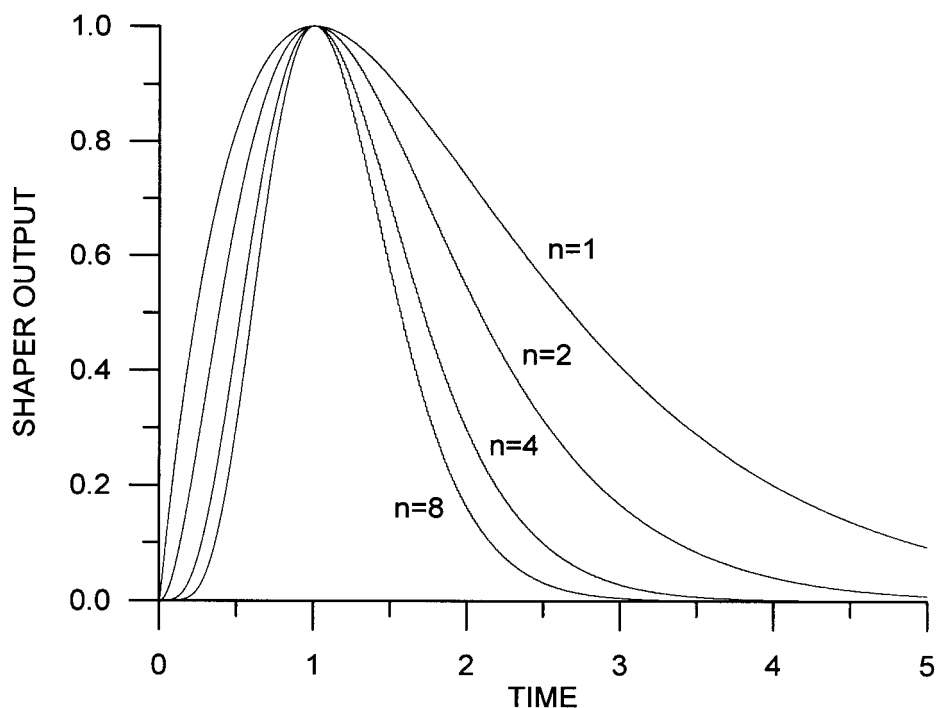
Output attains 90% of input slope after  $t = 2.3\tau$ .

Delay for  $n$  integrators =  $n\tau$

## Output pulse shape for multiple $RC$ integrators

Time constants changed to preserve the peaking time

$$(\tau_n = \tau_{n=1} / n)$$



Increasing the number of integrators makes the output pulse more symmetrical with a faster return to baseline.

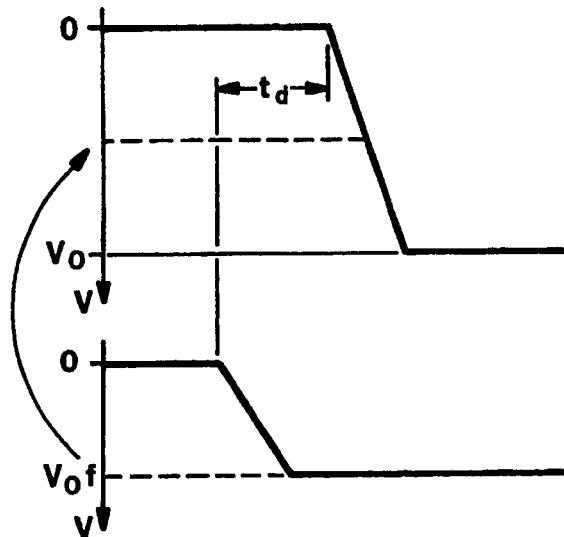
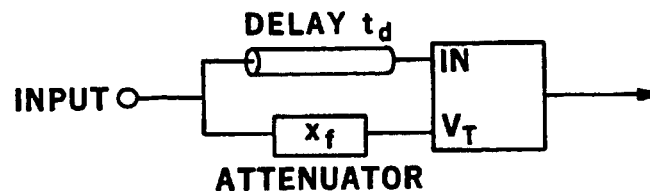
⇒ improved rate capability at the same peaking time

but ... increased curvature at beginning of pulse limits minimum threshold in timing measurements

## Constant Fraction Timing

Basic Principle:

make the threshold track the signal



The threshold is derived from the signal by passing it through an attenuator  $V_T = f V_s$ .

The signal applied to the comparator input is delayed so that the transition occurs after the threshold signal has reached its maximum value  $V_T = f V_0$ .



For simplicity assume a linear leading edge

$$V(t) = \frac{t}{t_r} V_0 \quad \text{for } t \leq t_r \quad \text{and} \quad V(t) = V_0 \quad \text{for } t > t_r$$

so the signal applied to the input is

$$V(t) = \frac{t - t_d}{t_r} V_0$$

When the input signal crosses the threshold level

$$fV_0 = \frac{t - t_d}{t_r} V_0$$

and the comparator fires at the time

$$t = f t_r + t_d \quad (t_d > t_r)$$

at a constant fraction of the rise time independent of peak amplitude.

If the delay  $t_d$  is reduced so that the pulse transitions at the signal and threshold inputs overlap, the threshold level

$$V_T = f \frac{t}{t_r} V_0$$

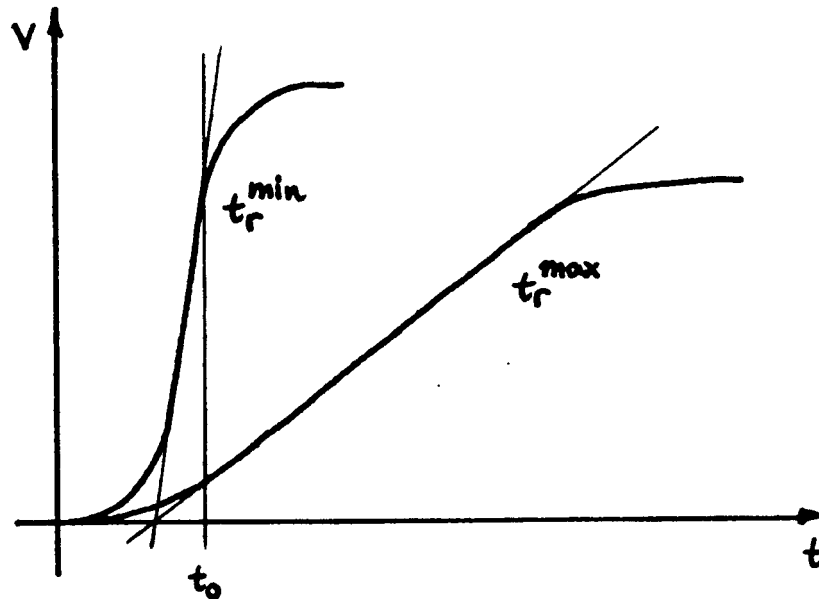
and the comparator fires at

$$f \frac{t}{t_r} V_0 = \frac{t - t_d}{t_r} V_0$$

$$t = \frac{t_d}{1 - f} \quad (t_d < (1 - f) t_r)$$

independent of both amplitude and rise time (amplitude and rise-time compensation).

The circuit compensates for amplitude and rise time if pulses have a sufficiently large linear range that extrapolates to the same origin.



The condition for the delay must be met for the minimum rise time:

$$t_d \leq (1 - f) t_{r,\min}$$

In this mode the fractional threshold  $V_T/V_0$  varies with rise time.

For all amplitudes and rise times within the compensation range the comparator fires at the time

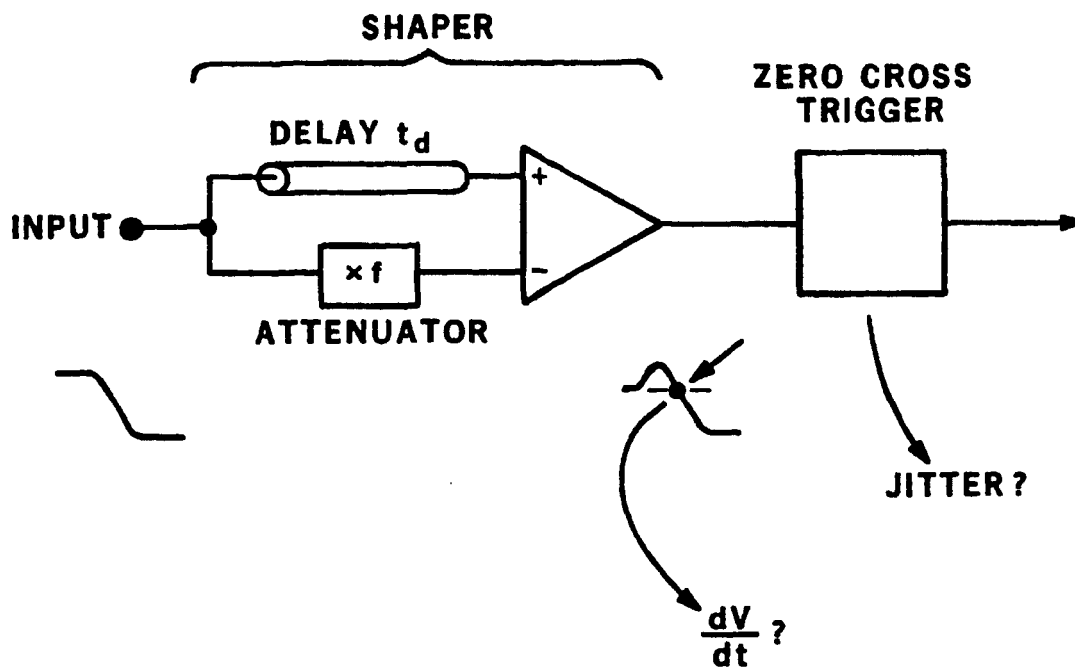
$$t_0 = \frac{t_d}{1 - f}$$

## Another View of Constant Fraction Discriminators

The constant fraction discriminator can be analyzed as a pulse shaper, comprising the

- delay
- attenuator
- subtraction

driving a trigger that responds to the zero crossing.

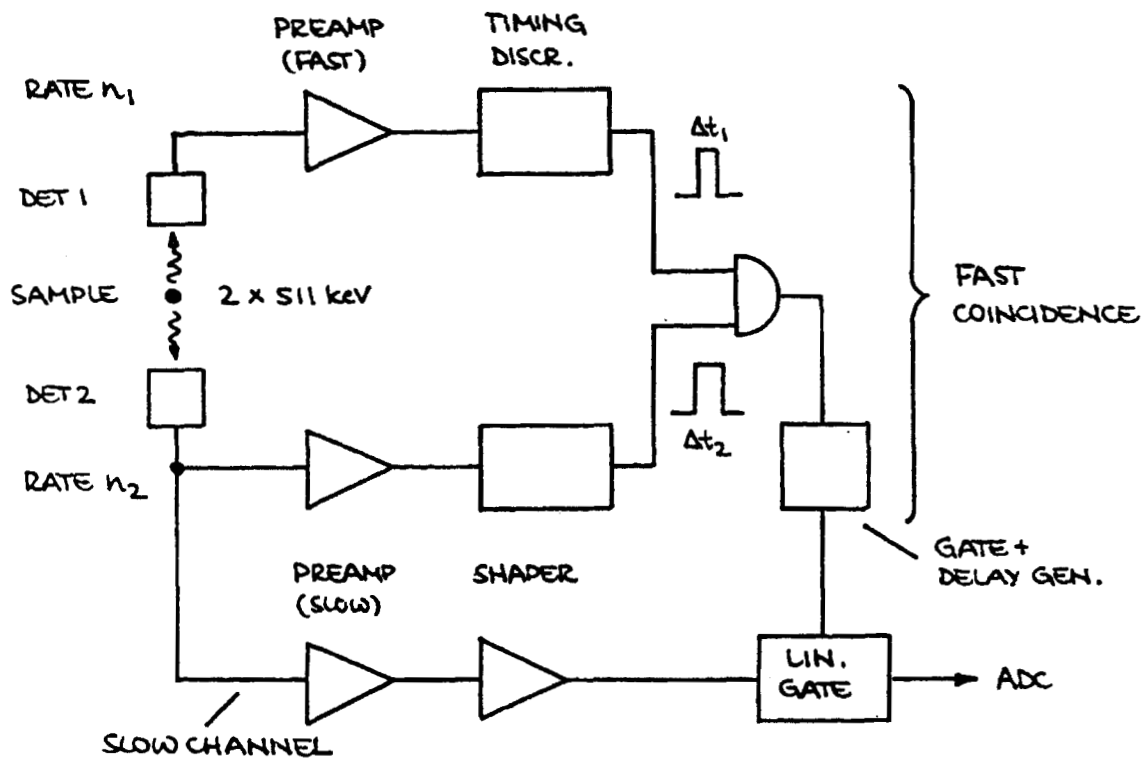


The timing jitter depends on

- the slope at the zero-crossing  
(depends on choice of  $f$  and  $t_d$ )
- the noise at the output of the shaper  
(this circuit increases the noise bandwidth)

## Examples

### 1. $\gamma$ - $\gamma$ coincidence (as used in positron emission tomography)



Positron annihilation emits two collinear 511 keV photons.

Each detector alone will register substantial background.

Non-coincident background can be suppressed by requiring simultaneous signals from both detectors.

- Each detector feeds a fast timing channel.
- The timing pulses are combined in an AND gate (coincidence unit). The AND gate only provides an output if the two timing pulses overlap.
- The coincidence output is used to open a linear gate, that allows the energy signal to pass to the ADC.

This arrangement accommodates the contradictory requirements of timing and energy measurements. The timing channels can be fast, whereas the energy channel can use slow shaping to optimize energy resolution (“fast-slow coincidence”).

### Chance coincidence rate

Two random pulse sequences have some probability of coincident events.

If the event rates in the two channels are  $n_1$  and  $n_2$ , and the timing pulse widths are  $\Delta t_1$  and  $\Delta t_2$ , the probability of a pulse from the first source occurring in the total coincidence window is

$$P_1 = n_1 \cdot (\Delta t_1 + \Delta t_2)$$

The coincidence is “sampled” at a rate  $n_2$ , so the chance coincidence rate is

$$n_c = P_1 \cdot n_2$$

$$n_c = n_1 \cdot n_2 \cdot (\Delta t_1 + \Delta t_2)$$

i.e. in the arrangement shown above, the chance coincidence rate increases with the square of the source strength.

Example:  $n_1 = n_2 = 10^6 \text{ s}^{-1}$

$$\Delta t_1 = \Delta t_2 = 5 \text{ ns}$$

$$\Rightarrow n_c = 10^4 \text{ s}^{-1}$$

## 2. Nuclear Mass Spectroscopy by Time-of-Flight

Two silicon detectors

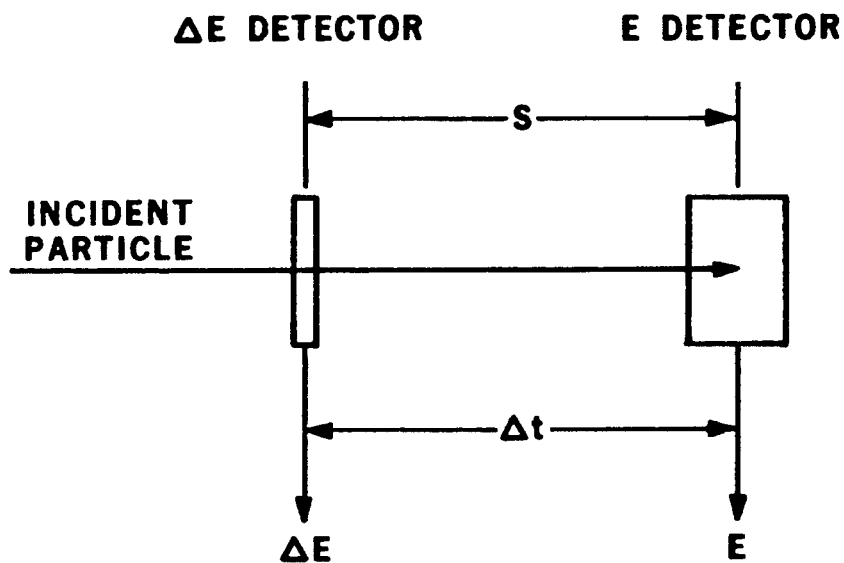
First detector thin, so that particle passes through it  
(transmission detector)

⇒ differential energy loss  $\Delta E$

Second detector thick enough to stop particle

⇒ Residual energy  $E$

Measure time-of-flight  $\Delta t$  between the two detectors



$$\Delta E + E \rightarrow E_{TOT}$$

$$\Delta E, E \rightarrow Z$$

$$\Delta t, E \rightarrow A$$

$$E_{tot} = \Delta E + E$$

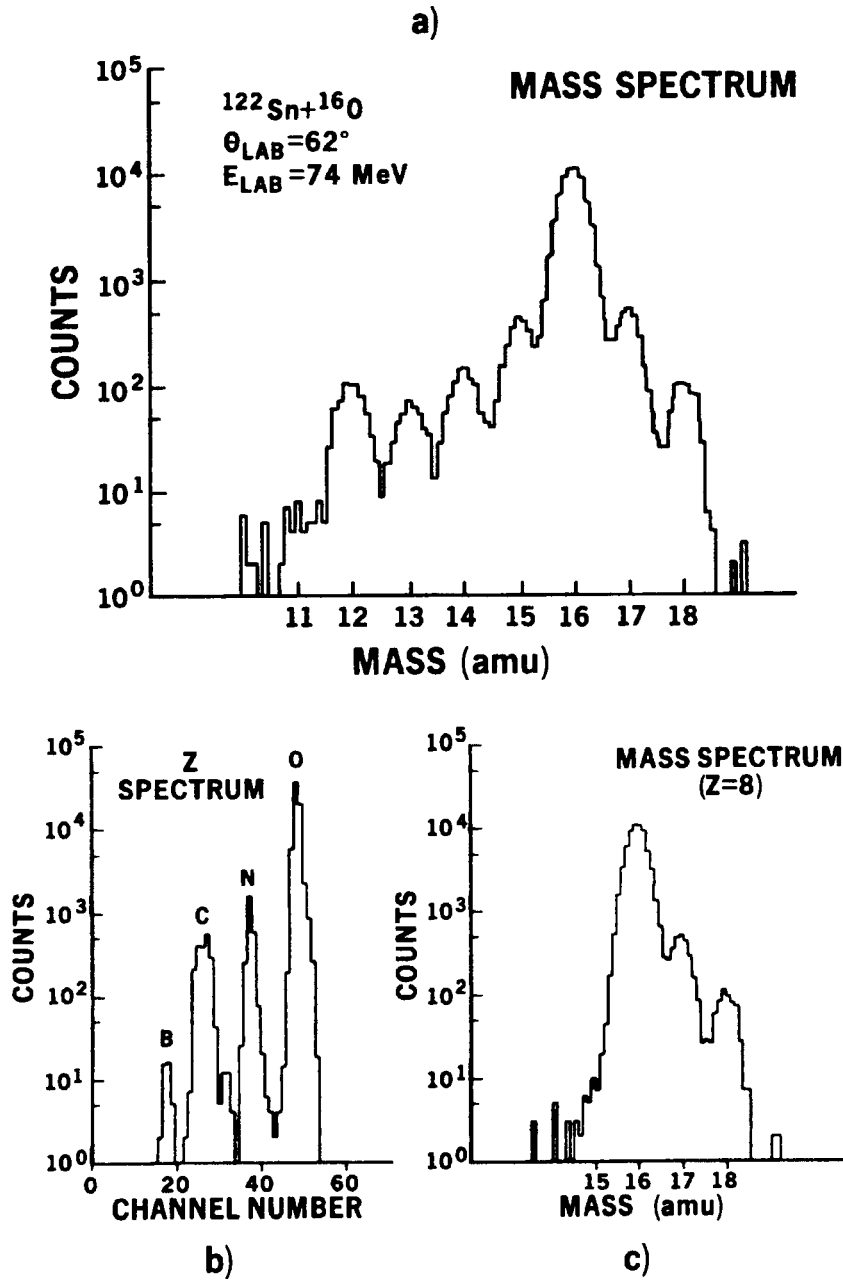
$$Z \propto \sqrt{\Delta E \cdot E_{tot}}$$

$$A \propto E \cdot (\Delta t / s)^2$$

# “Typical” Results

## Example 1

Flight path 20 cm,  $\Delta t \approx 50$  ps FWHM  
 $\sigma_t = 21$  ps



(H. Spieler et al., Z. Phys. **A278** (1976) 241)

### Example 2

1.  $\Delta E$ -detector: 27  $\mu\text{m}$  thick,  $A = 100 \text{ mm}^2$ ,  $\langle E \rangle = 1.1 \cdot 10^4 \text{ V/cm}$

2.  $E$ -detector: 142  $\mu\text{m}$  thick,  $A = 100 \text{ mm}^2$ ,  $\langle E \rangle = 2 \cdot 10^4 \text{ V/cm}$

For 230 MeV  $^{28}\text{Si}$ :  $\Delta E = 50 \text{ MeV} \Rightarrow V_s = 5.6 \text{ mV}$

$E = 180 \text{ MeV} \Rightarrow V_s = 106 \text{ mV}$

$\Rightarrow \Delta t = 32 \text{ ps FWHM}$

$\sigma_t = 14 \text{ ps}$

### Example 3

Two transmission detectors,

each 160  $\mu\text{m}$  thick,  $A = 320 \text{ mm}^2$

For 650 MeV/u  $^{20}\text{Ne}$ :  $\Delta E = 4.6 \text{ MeV} \Rightarrow V_s = 800 \mu\text{V}$

$\Rightarrow \Delta t = 180 \text{ ps FWHM}$

$\sigma_t = 77 \text{ ps}$

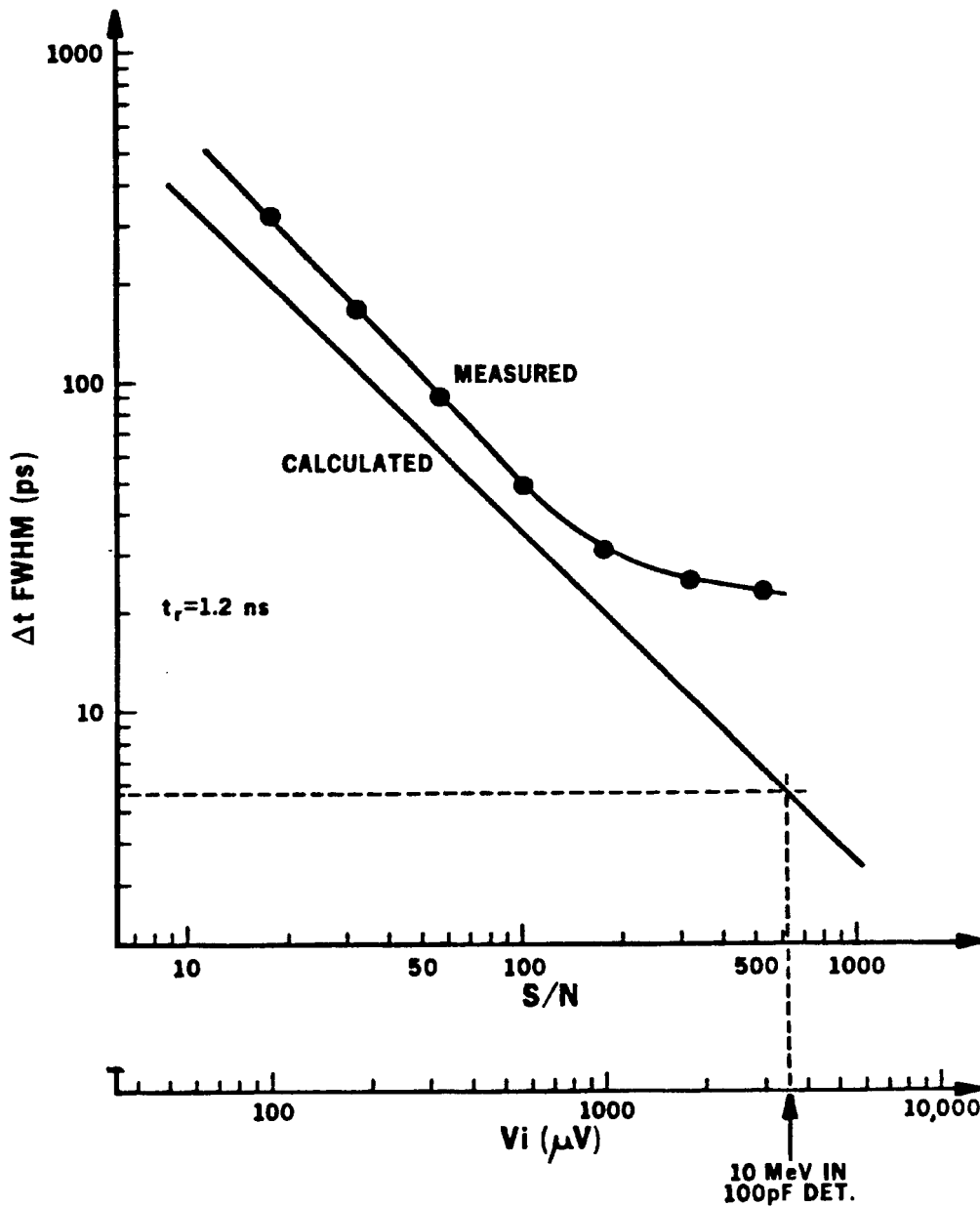
For 250 MeV/u  $^{20}\text{Ne}$ :  $\Delta E = 6.9 \text{ MeV} \Rightarrow V_s = 1.2 \text{ mV}$

$\Rightarrow \Delta t = 120 \text{ ps FWHM}$

$\sigma_t = 52 \text{ ps}$



## Fast Timing: Comparison between theory and experiment



At  $S/N < 100$  the measured curve lies above the calculation because the timing discriminator limited the rise time.

At high  $S/N$  the residual jitter of the time digitizer limits the resolution.

For more details on fast timing with semiconductor detectors, see H. Spieler, IEEE Trans. Nucl. Sci. **NS-29/3** (1982) 1142.

## Timing with Photomultiplier Tubes

### 1. Scintillator

Assume a scintillator with a single decay constant  $\tau$ , which coupled to a photomultiplier tube yields a total number of photoelectrons  $N$ .

The probability of the emission of a single photoelectron in the time interval  $t, t+dt$  is

$$P(t) = \frac{1}{\tau} e^{-t/\tau} dt$$

The mean emission rate is

$$\frac{d\bar{N}}{dt} = \frac{\bar{N}}{\tau} e^{-t/\tau}$$

The probability that the  $n$ th photon is emitted in the time interval  $t, t+dt$  is

$$P_n(t)dt = \frac{[f(t)]^{n-1} e^{-f(t)} [df(t)/dt] dt}{(n-1)!}$$

where  $f(t)$  is the average or expected number of photons emitted up to the time  $t$ . (R.F. Post and L.I. Schiff, Phys. Rev. **80** (1950) 1113)

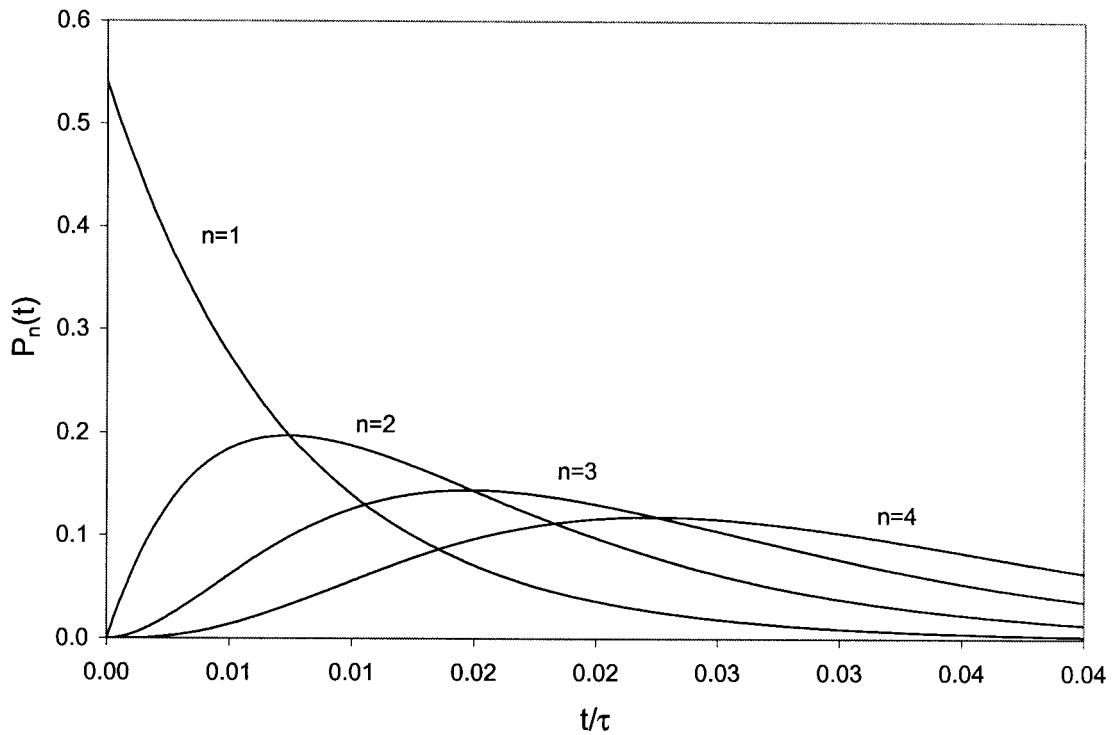
For the simple exponential decay shown above

$$f(t) = \bar{N}(1 - e^{-t/\tau})$$

and

$$P_n(t) = \frac{\bar{N}^n (1 - e^{-t/\tau})^{n-1} \exp[-\bar{N}(1 - e^{-t/\tau})] e^{-t/\tau}}{\tau (n-1)!}$$

Time distributions for triggering on the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> photoelectron



The time resolution is best for  $Q = 1$ , which at small times yields an exponential timing distribution with the time constant  $\tau / \bar{N}$

$$P_1(t) = \frac{\bar{N}}{\tau} e^{-t/\tau} \exp[-\bar{N}(1 - e^{-t/\tau})] \approx \frac{\bar{N}}{\tau} e^{-\bar{N}t/\tau}$$

(F. Lynch, IEEE Trans. Nucl. Sci., NS-13/3 (1966) 140)

## 2. The Photomultiplier

Photoelectrons emitted from the photocathode are subject to time variations in reaching the anode.

For a typical fast 2" PMT (Philips XP2020) the transit time from the photocathode to the anode is about 30 ns at 2000V.

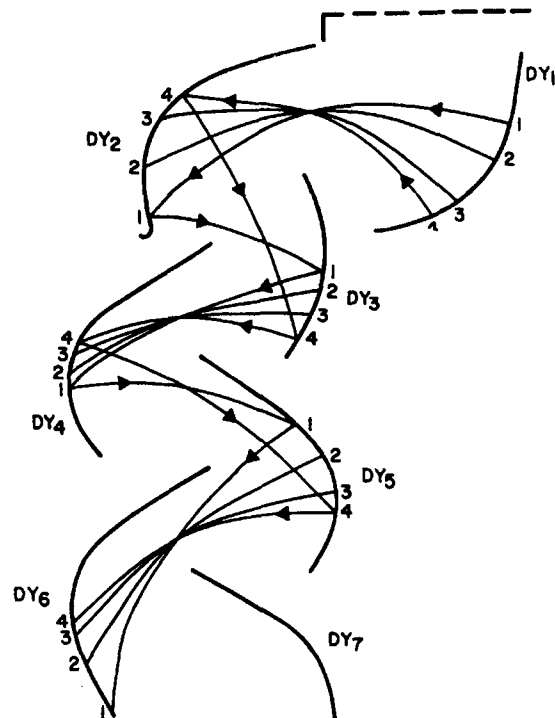
The intrinsic rise time is 1.6 ns, due to broadening of the initial electron packet in the course of the multiplication process.

The transit time varies by 0.25 ns between the center of the photocathode and a radius of 18 mm.

For two tubes operating in coincidence at a signal level of 1500 photoelectrons, a time resolution of 230 ps is possible.

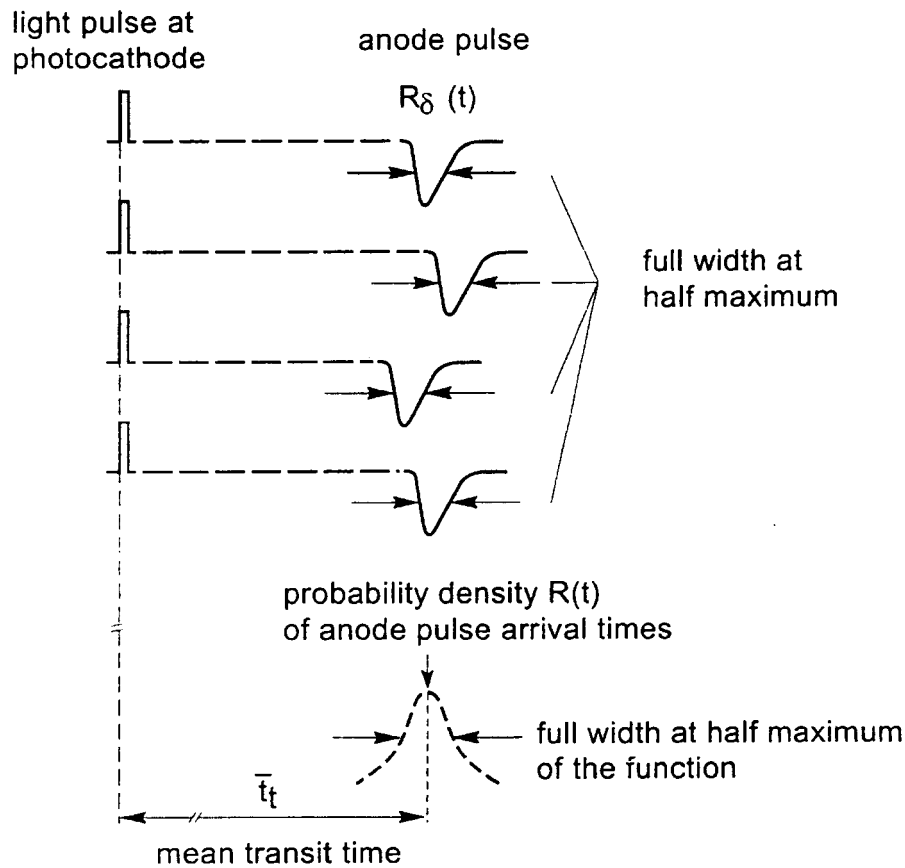
Special dynode structures are used to reduce transit time spread.

Example: time compensating structure



(from *Burle Photomultiplier Handbook*)

Even for photons impinging on a given position, the transit time through a photomultiplier varies from photon to photon, since photoelectrons are emitted from the photocathode with varying velocities and directions.



(from *Photomultiplier Tubes*, Philips Photonics)

The transit time “jitter” distribution is often gaussian, yielding an instantaneous anode current

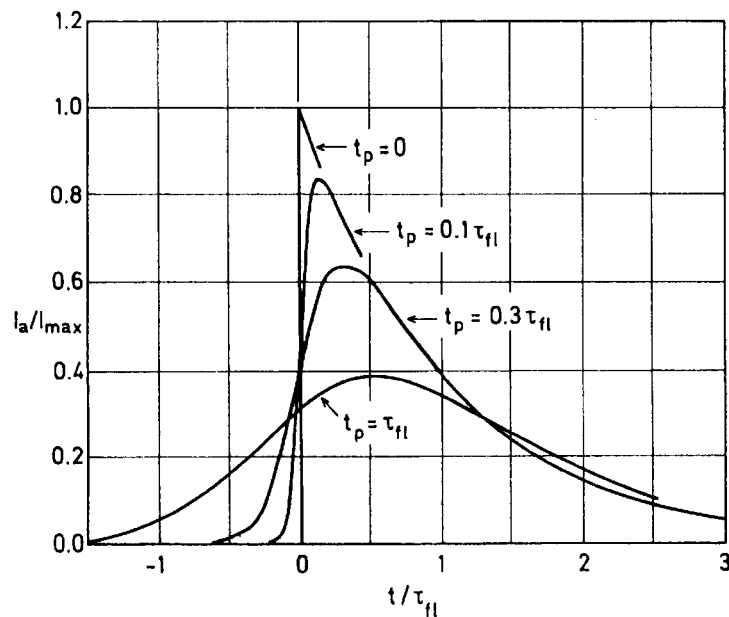
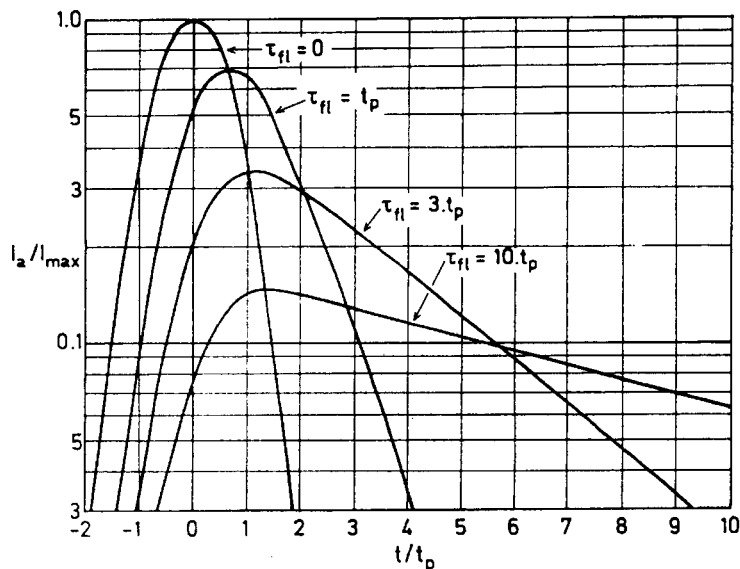
$$i_a(t) = \frac{\bar{A}e}{t_p \sqrt{\pi}} e^{-(t/t_p)^2}$$

Then the response to the scintillator becomes the convolution of the exponential decay function with the gaussian transit time spread

$$I_a(t) = \frac{\bar{A}e\bar{N}}{\tau t_p} \cdot \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-[(t-t')/t_p]^2} e^{-t'/\tau} dt'$$

The transit time spread imparts a finite rise time to the output pulse, due to the smearing of the arrival times of electrons at the anode.

PMT output pulses for various values of scintillator decay time  $\tau_{fl}$  and transit time jitter  $t_p$ .



(from Kowalski, *Nuclear Electronics*)

If the decay time of the scintillator is short with respect to the transit time spread, signals from successive photoelectrons will be comingled and the signal from the first photon can no longer be distinguished from the response to successive photons.

Now it may become advantageous to trigger after some integration time. As a consequence, fast scintillators often provide the best timing at a fixed fraction of their peak output signal, typically 0.1 to 0.3. Only for relative slow scintillators, NaI(Tl) with a decay time of 250 ns for example, is the fraction very small, of order 1%.

Many scintillators exhibit an inherent rise time

$$N(t) = \frac{e^{-t/\tau_2} - e^{-t/\tau_1}}{\tau_2 - \tau_1}$$

Here,  $\tau_1$  represents the non-radiative transitions that feed the optically active states, which emit photons with the time constant  $\tau_2$ .

This expression is accurate for binary solution scintillators and is a good approximation for ternary solution scintillators. Here triggering at 10 to 30% of the peak pulse height is nearly always advantageous, since the rise time of the scintillator masks the transit time spread of the photomultiplier.

## Digitization of Pulse Height – Analog to Digital Conversion

For data storage and subsequent analysis the analog signal at the shaper output must be digitized.

Important parameters for ADCs used in detector systems:

1. Resolution  
The “granularity” of the digitized output
2. Differential Non-Linearity  
How uniform are the digitization increments?
3. Integral Non-Linearity  
Is the digital output proportional to the analog input?
4. Conversion Time  
How much time is required to convert an analog signal to a digital output?
5. Count-Rate Performance  
How quickly can a new conversion commence after completion of a prior one without introducing deleterious artifacts?
6. Stability  
Do the conversion parameters change with time?

Instrumentation ADCs used in industrial data acquisition and control systems share most of these requirements. However, detector systems place greater emphasis on differential non-linearity and count-rate performance. The latter is important, as detector signals often occur randomly, in contrast to measurement systems where signals are sampled at regular intervals.



## 1. Resolution

Digitization incurs approximation, as a continuous signal distribution is transformed into a discrete set of values. To reduce the additional errors (noise) introduced by digitization, the discrete digital steps must correspond to a sufficiently small analog increment.

Simplistic assumption:

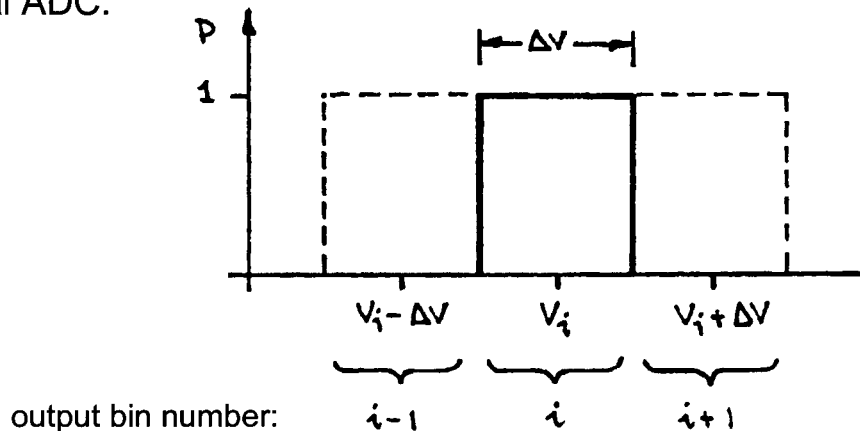
Resolution is defined by the number of output bits, e.g.

$$13 \text{ bits} \rightarrow \frac{\Delta V}{V} = \frac{1}{8192} = 1.2 \cdot 10^{-4}$$

True Measure: Channel Profile

Plot probability vs. pulse amplitude that a pulse height corresponding to a specific output bin is actually converted to that address.

Ideal ADC:



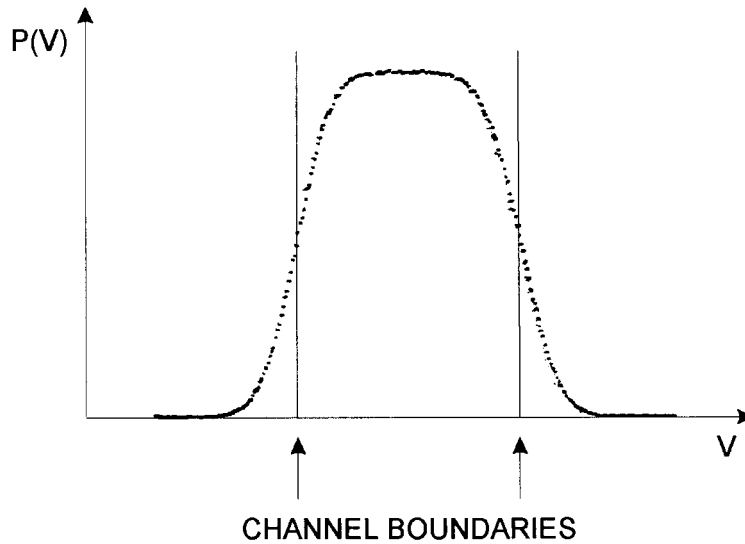
Measurement accuracy:

- If all counts of a peak fall in one bin, the resolution is  $\Delta V$ .
- If the counts are distributed over several ( $>4$  or  $5$ ) bins, peak fitting can yield a resolution of  $10^{-1} - 10^{-2} \Delta V$ .

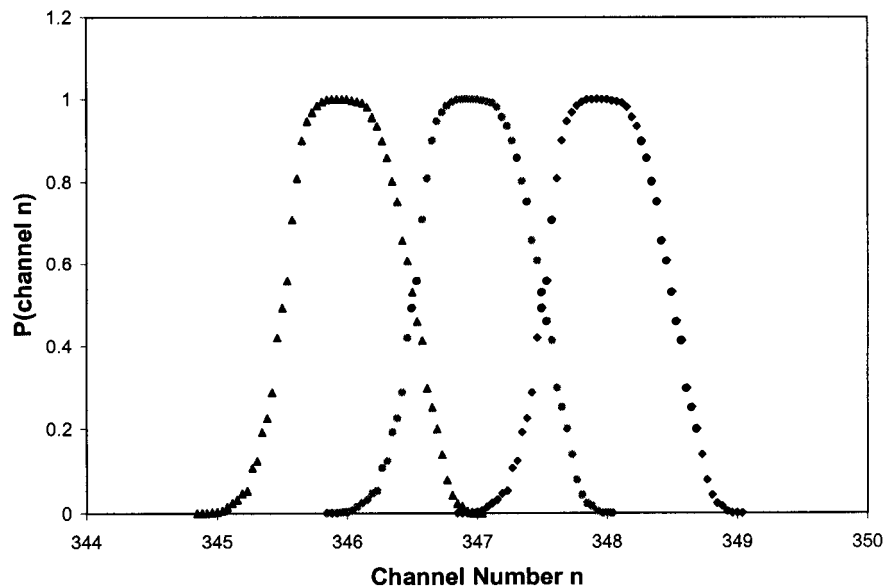
In reality, the channel profile is not rectangular as sketched above.

Electronic noise in the threshold discrimination process that determines the channel boundaries “smears” the transition from one bin to the next.

Measured channel profile (13 bit ADC)



The profiles of adjacent channels overlap



## 2. Differential Non-Linearity

Differential non-linearity is a measure of the inequality of channel profiles over the range of the ADC.

Depending on the nature of the distribution, either a peak or an rms specification may be appropriate.

$$DNL = \max \left\{ \frac{\Delta V(i)}{\langle \Delta V \rangle} - 1 \right\}$$

or

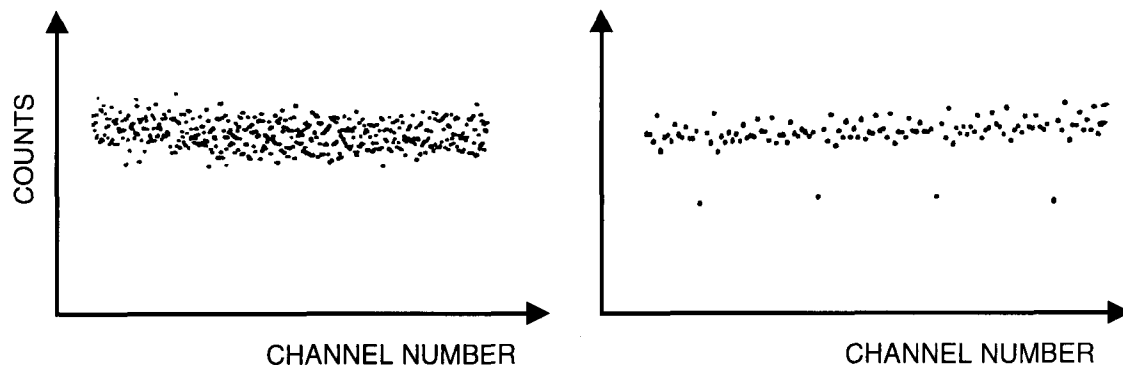
$$DNL = \text{r.m.s.} \left\{ \frac{\Delta V(i)}{\langle \Delta V \rangle} - 1 \right\}$$

where  $\langle \Delta V \rangle$  is the average channel width and  $\Delta V(i)$  is the width of an individual channel.

Differential non-linearity of  $< \pm 1\%$  max. is typical, but state-of-the-art ADCs can achieve  $10^{-3}$  rms, i.e. the variation is comparable to the statistical fluctuation for  $10^6$  random counts.

Note: Instrumentation ADCs are often specified with an accuracy of  $\pm 0.5$  LSB (least significant bit), so the differential non-linearity may be 50% or more.

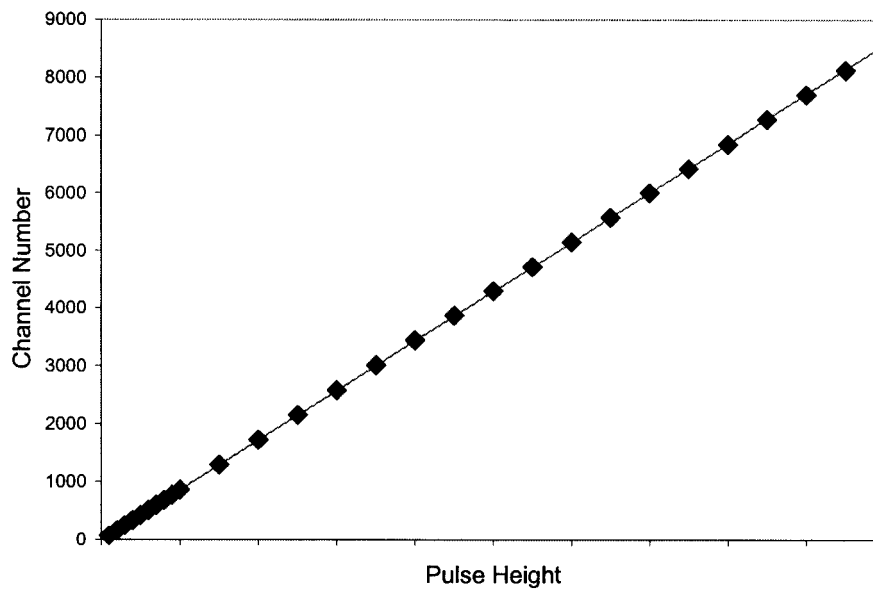
Typical differential non-linearity patterns (“white” input spectrum).  
An ideal ADC would show an equal number of counts in each bin.



The spectrum to the left shows a random pattern, but note the multiple periodicities visible in the right hand spectrum.

### 3. Integral Non-Linearity

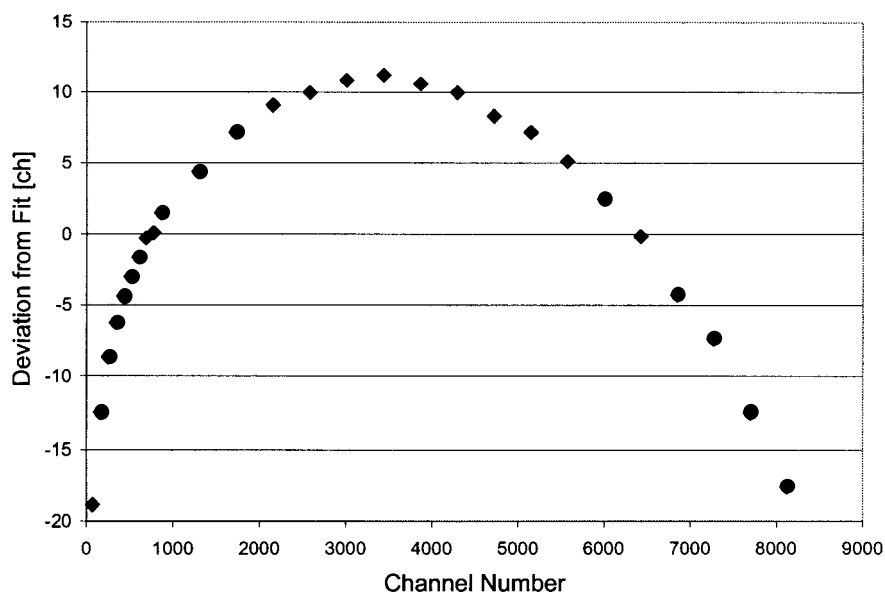
Integral non-linearity measures the deviation from proportionality of the measured amplitude to the input signal level.



The dots are measured values and the line is a fit to the data.

This plot is not very useful if the deviations from linearity are small.

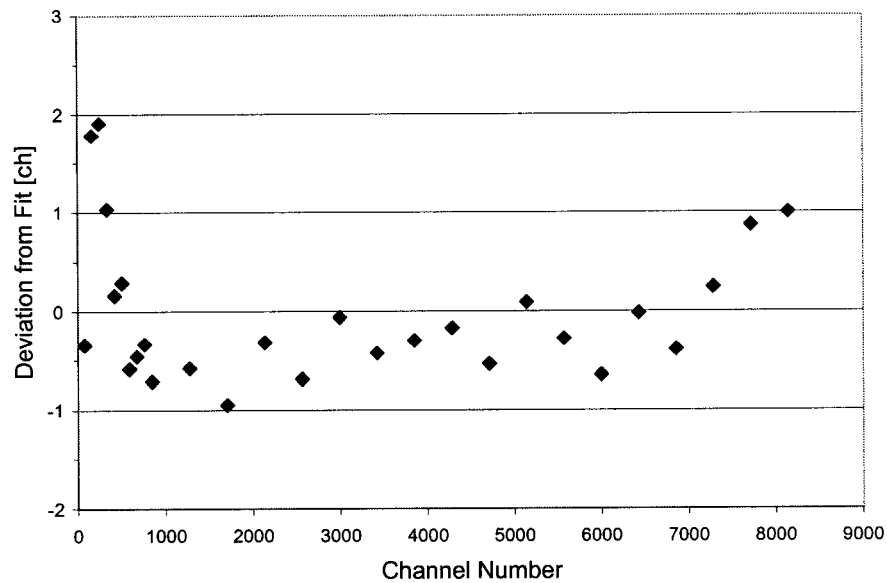
Plotting the deviations of the measured points from the fit yields:



The linearity of an ADC can depend on the input pulse shape and duration, due to bandwidth limitations in the circuitry.

The differential non-linearity shown above was measured with a 400 ns wide input pulse.

Increasing the pulse width to 3  $\mu$ s improved the result significantly:



#### 4. Conversion Time

During the acquisition of a signal the system cannot accept a subsequent signal (“dead time”)

Dead Time =

- signal acquisition time → time-to-peak + const.
- + conversion time → can depend on pulse height
- + readout time to memory → depends on speed of data transmission and buffer memory access - can be large in computer-based systems

Dead time affects measurements of yields or reaction cross-sections. Unless the event rate  $\ll 1/(\text{dead time})$ , it is necessary to measure the dead time, e.g. with a reference pulser fed simultaneously into the spectrum.

The total number of reference pulses issued during the measurement is determined by a scaler and compared with the number of pulses recorded in the spectrum.

Does a pulse-height dependent dead time mean that the correction is a function of pulse height?

Usually not. If events in different part of the spectrum are not correlated in time, i.e. random, they are all subject to the same average dead time (although this average will depend on the spectral distribution).

- Caution with correlated events!  
 Example: Decay chains, where lifetime is  $<$  dead time.  
 The daughter decay will systematically be lost.

## 5. Count Rate Effects

Problems are usually due to internal baseline shifts with event rate or undershoots following a pulse.

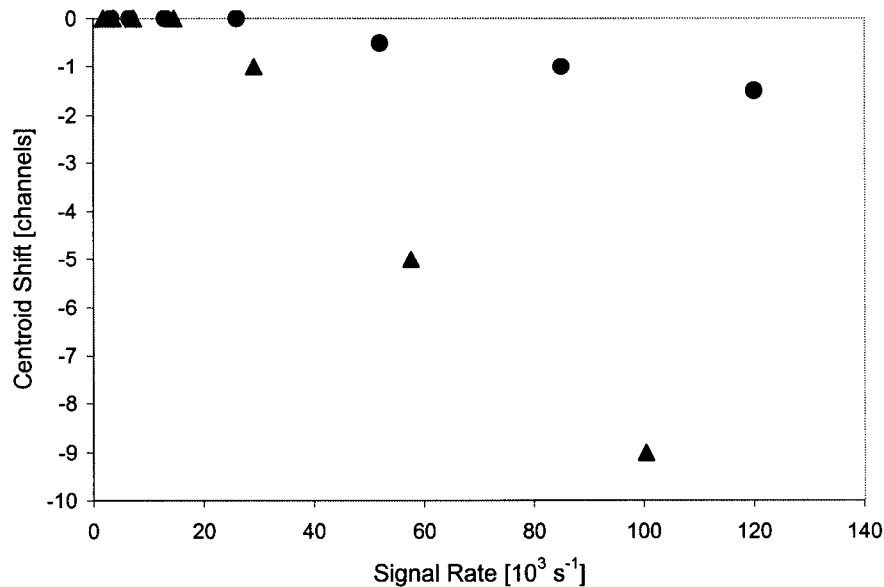
If signals occur at constant intervals, the effect of an undershoot will always be the same.

However, in a random sequence of pulses, the effect will vary from pulse to pulse.

⇒ spectral broadening

Baseline shifts tend to manifest themselves as a systematic shift in centroid position with event rate.

Centroid shifts for two 13 bit ADCs vs. random rate:



## 6. Stability

Stability vs. temperature is usually adequate with modern electronics in a laboratory environment.

- Note that temperature changes within a module are typically much smaller than ambient.

However: Highly precise or long-term measurements require spectrum stabilization to compensate for changes in gain and baseline of the overall system.

Technique: Using precision pulsers place a reference peak at both the low and high end of the spectrum.

(Pk. Pos. 2) – (Pk. Pos. 1) → Gain, ... then

(Pk. Pos. 1) or (Pk. Pos. 2) → Offset

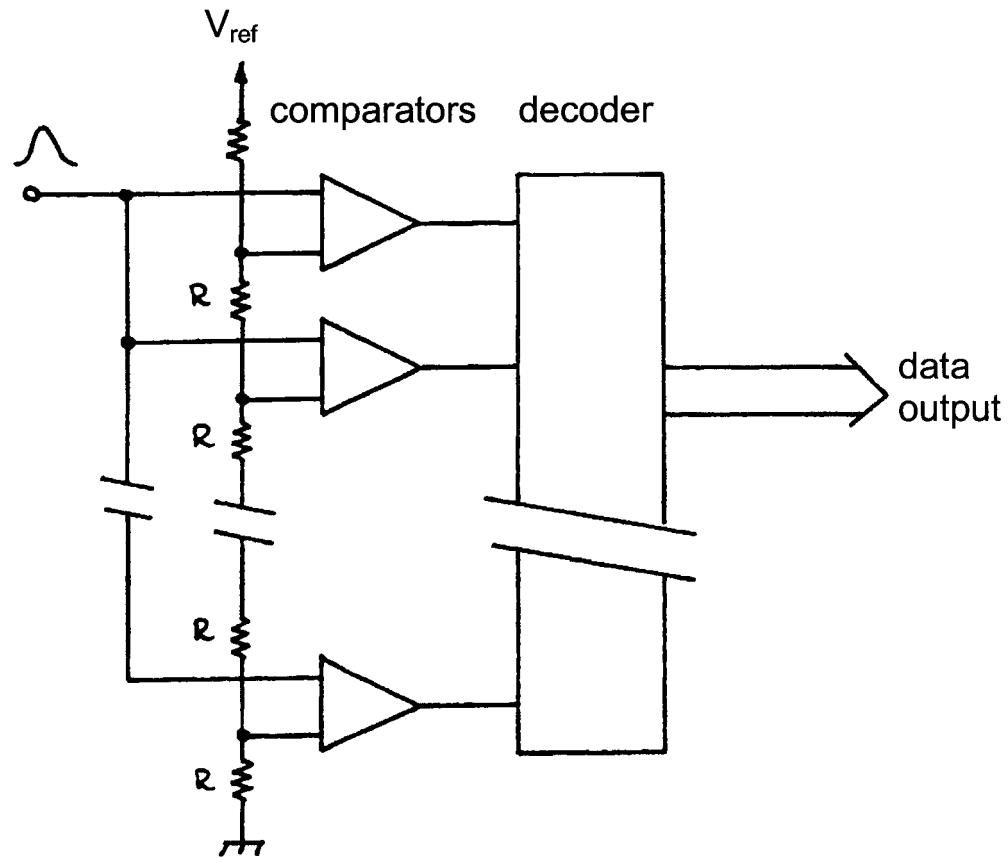
Traditional Implementation: Hardware,  
spectrum stabilizer module

Today, it is more convenient to determine the corrections in software. These can be applied to calibration corrections or used to derive an electrical signal that is applied to the hardware (simplest and best in the ADC).



# Analog to Digital Conversion Techniques

## 1. Flash ADC



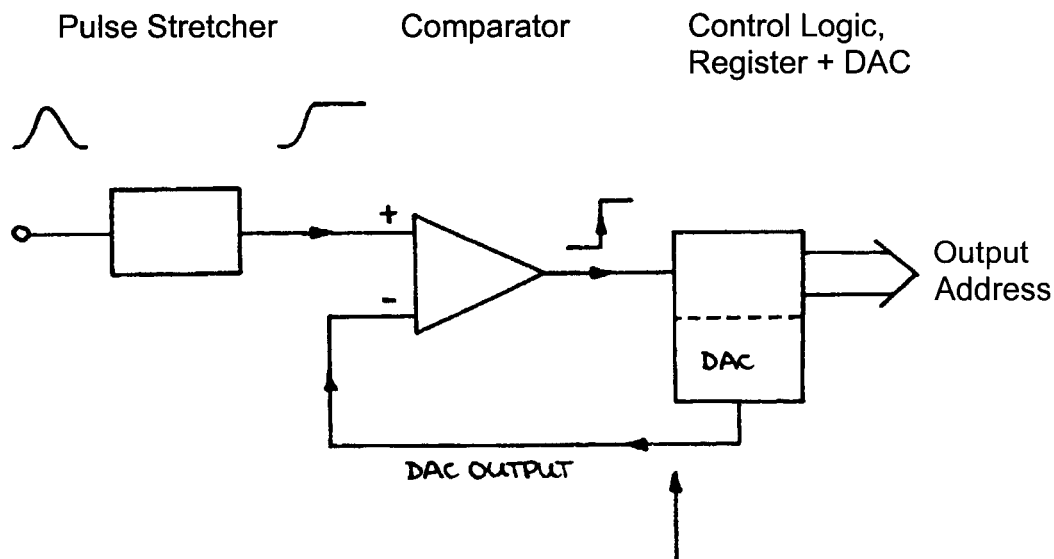
The input signal is applied to  $n$  comparators in parallel. The switching thresholds are set by a resistor chain, such that the voltage difference between individual taps is equal to the desired measurement resolution.

$2^n$  comparators for  $n$  bits (8 bit resolution requires 256 comparators)

Feasible in monolithic ICs since the absolute value of the resistors in the reference divider chain is not critical, only the relative matching.

Advantage: short conversion time (<10 ns available)  
 Drawbacks: limited accuracy (many comparators)  
 power consumption  
 Differential non-linearity  $\sim 1\%$   
 High input capacitance (speed is often limited by the analog driver feeding the input)

## 2. Successive Approximation ADC



Sequentially add levels proportional to  $2^n, 2^{n-1}, \dots 2^0$  and set corresponding bit if the comparator output is high (DAC output < pulse height)

n conversion steps yield  $2^n$  channels,  
i.e. 8K channels require 13 steps

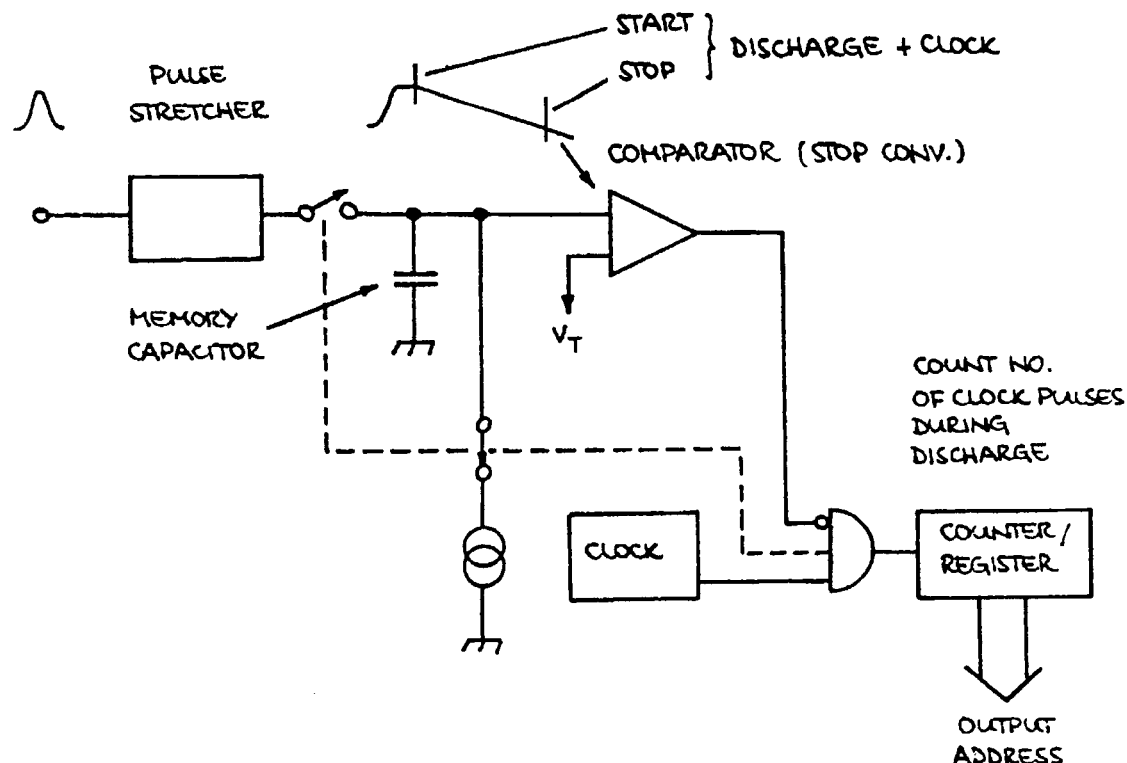
Advantages: speed ( $\sim \mu\text{s}$ )  
high resolution  
ICs (monolithic + hybrid) available

Drawback: Differential non-linearity (typ. 10 – 20%)

Reason: Resistors that set DAC output must be extremely accurate.

For DNL < 1% the resistor determining the  $2^{12}$  level in an 8K ADC must be accurate to  $< 2.4 \cdot 10^{-6}$ .

### 3. Wilkinson ADC



The peak signal amplitude is acquired by a pulse stretcher and transferred to a memory capacitor. Then, simultaneously,

1. the capacitor is disconnected from the stretcher,
2. a current source is switched to linearly discharge the capacitor,
3. a counter is enabled to determine the number of clock pulses until the voltage on the capacitor reaches the baseline.

Advantage: excellent differential linearity  
(continuous conversion process)

Drawbacks: slow – conversion time =  $n \cdot T_{clock}$   
( $n$  = channel number  $\propto$  pulse height)

$$T_{clock} = 10 \text{ ns} \rightarrow T_{conv} = 82 \mu\text{s} \text{ for 13 bits}$$

Clock frequencies of 100 MHz typical, but  
>400 MHz possible with excellent performance

“Standard” technique for high-resolution spectroscopy.

## Hybrid Analog-to-Digital Converters

Conversion techniques can be combined to obtain high resolution and short conversion time.

### 1. Flash + Successive Approximation or Flash + Wilkinson (Ramp Run-Down)

Utilize fast flash ADC for coarse conversion  
(e.g. 8 out of 13 bits)

Successive approximation or Wilkinson converter to provide fine resolution. Limited range, so short conversion time:

256 ch with 100 MHz clock  $\Rightarrow$  2.6  $\mu$ s

Results: 13 bit conversion in  $< 4 \mu$ s  
with excellent integral and differential linearity

### 2. Flash ADCs with Sub-Ranging

Not all applications require constant absolute resolution over the full range. Sometimes only *relative* resolution must be maintained, especially in systems with a very large dynamic range.

Precision binary divider at input to determine coarse range + fast flash ADC for fine digitization.

Example: Fast digitizer that fits in phototube base.  
Designed at FNAL.

17 to 18 bit dynamic range  
Digital floating point output  
(4 bit exponent, 8+1 bit mantissa)  
16 ns conversion time

## Time Digitizers

### 1. Counter

Simplest arrangement.

Count clock pulses between start and stop.

Limitation: Speed of counter

Current technology limits speed of counter system to about 1 GHz

$$\Rightarrow \Delta t = 1 \text{ ns}$$

Multi-hit capability

### 2. Analog Ramp

Commonly used in high-resolution digitizers ( $\Delta t = 10 \text{ ps}$ )

Principle: charge capacitor through switchable current source

Start pulse: turn on current source

Stop pulse: turn off current source

$$\Rightarrow \text{Voltage on storage capacitor}$$

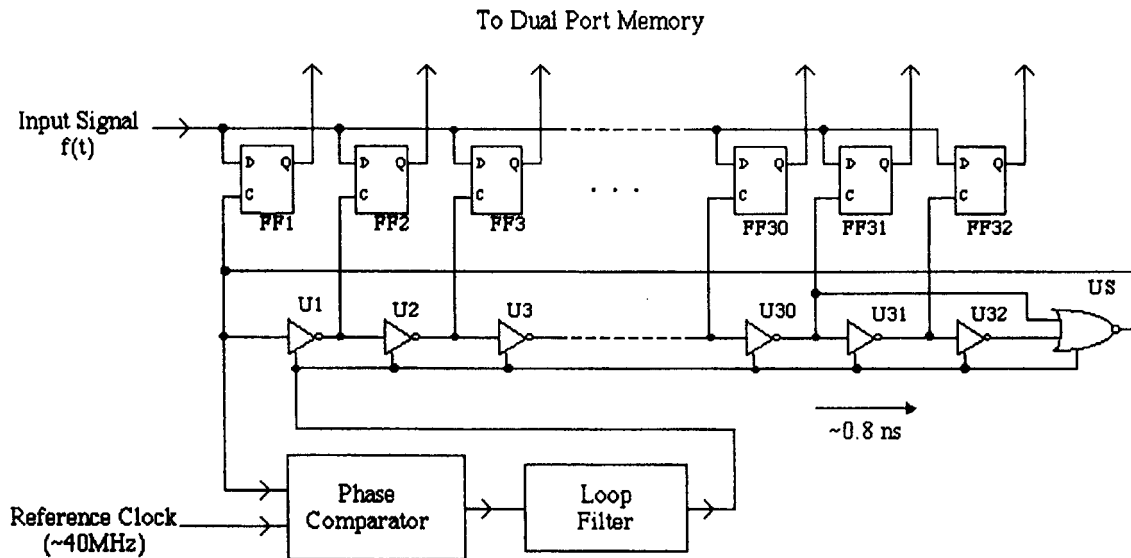
Use Wilkinson ADC with smaller discharge current to digitize voltage.

Drawbacks: No multi-hit capability  
Deadtime

### 3. Digitizers with Clock Interpolation

Most experiments in HEP require multi-hit capability, no deadtime

Commonly used technique for time digitization (Y. Arai, KEK)



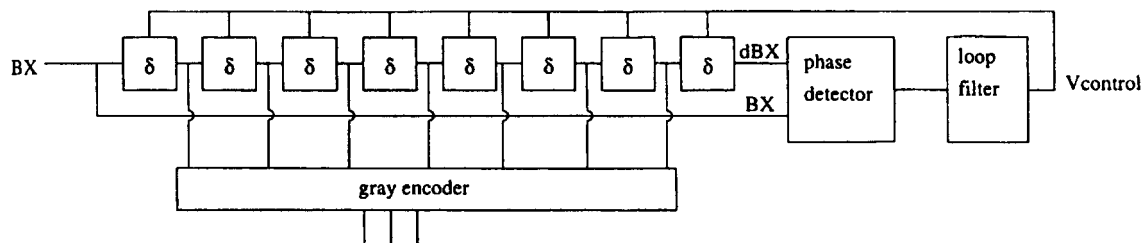
- Digitizing an external signal at a period of internal gate delay of  $\sim 0.8$  ns.
- High Stability with a Phase Locked Loop.
- Long time range ( $> 3$   $\mu$ s) & No deadtime by a Dual Port Memory.
- High precision, High Density & Low Cost LSI.

Patent Pending  
 ● S63-067314 (JP)  
 ● H3-133169 (JP)  
 ● H6-69507 (JP)  
 ● 95300652.5 (EU)

Clock period interpolated by inverter delays (U1, U2, ...).

Delay can be fine-tuned by adjusting operating point of inverters.

Delays stabilized by delay-locked loop



Devices with 250 ps resolution fabricated and tested.

see Y. Arai et al., IEEE Trans. Nucl. Sci. NS-45/3 (1998) 735-739  
 and references therein.