Interactive Tutorial on Fundamentals of Signal Integrity for High-Speed/High-Density Design

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Outline

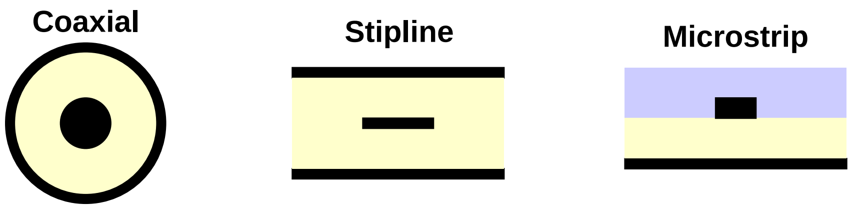
- **Introduction** **(A. Deutsch)**
  - The interconnect bottleneck in high-speed systems
- **Interconnect Modeling Fundamentals** **(A.Cangellaris/U. Ravaioli)**
  - Time-domain & frequency-domain transmission line analysis
  - Lossy lines and signal dispersion
  - Crosstalk for short lengths of coupled interconnects
- **On-Chip Interconnects** **(A. Deutsch)**
  - Modeling of on-chip interconnects
  - Interconnect impact on system performance
  - Future trends

Outline (cont.)

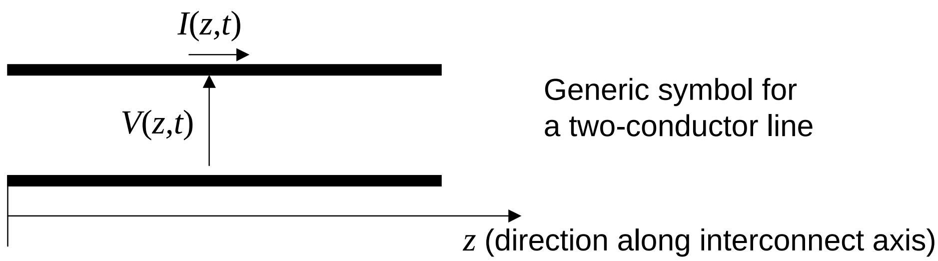
- **Interconnects at the Package and Board Level**
  (J.Schutt-Aine/U. Ravaioli)
  - Multiconductor transmission line theory
  - Crosstalk modeling and measurement
  - Lumped vs. distributed modeling of interconnects
- **Concluding Remarks**

Fundamentals of Transmission Line Theory

Transmission-line theory quantifies signal propagation on a system of two parallel conductors with cross-sectional dimensions much smaller than their length

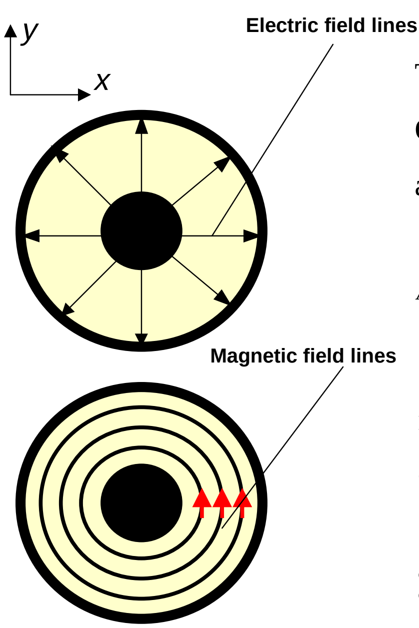


Familiar
Cross sections



Generic symbol for
a two-conductor line

For a uniform transmission line, the electric and magnetic fields are transverse to the direction of wave propagation (and hence, to the axis of the line). Thus, transmission line fields are called Transverse Electromagnetic (TEM) Waves



The electric field behaves like an electrostatic field.

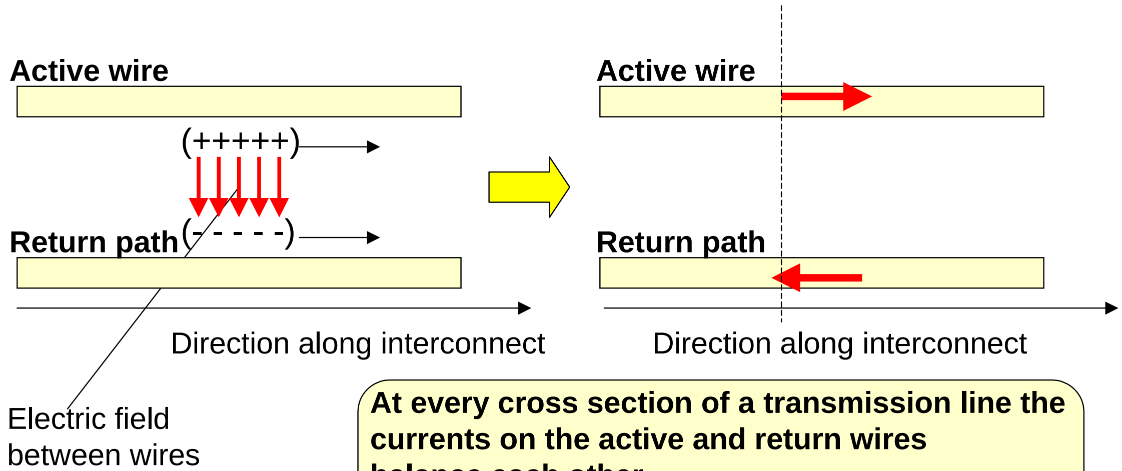
Over the cross section, the potential difference between any two points A and B on the two conductors is constant:

$$\int_{A\to B} \vec{E}(x,y,z,t)\cdot d\vec{l} = V(z,t)$$

Over the cross section, the magnetic field looks like a magnetostatic field. Its line integral around one of the conductors equals the total current in the conductor.

$$\oint_{\substack{\text{center}\\ \text{conductor}}} \vec{H}(x,y,z,t)\cdot d\vec{l} = I(z,t)$$

In a transmission line configuration as much charge moves down the “active” wire that much charge of negative polarity moves down the “return path”



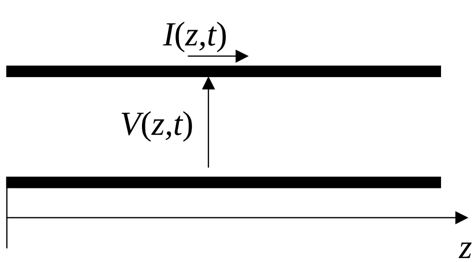
At every cross section of a transmission line the currents on the active and return wires balance each other.
This balance leads to field confinement and reduced interference.

Signal propagation is quantified in terms of the solution of the so-called Telegrapher’s equations

Time-Domain Form

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t}$$

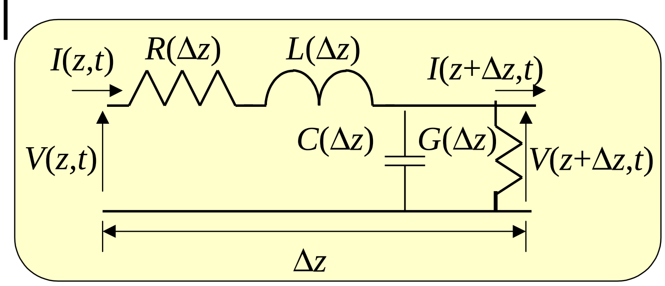
$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}$$



Frequency-Domain Form

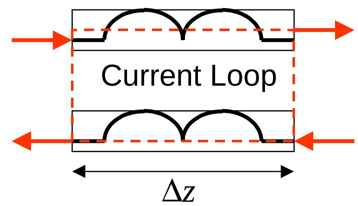
$$\frac{dV(z,\omega)}{dz} = -RI(z,\omega) - j\omega LI(z,\omega)$$

$$\frac{dI(z,\omega)}{dz} = -GV(z,\omega) - j\omega CV(z,\omega)$$



Transmission-Line Parameters

- Per-unit-length capacitance *C*
- Per-unit-length conductance G
- Per-unit-length inductance *L*
  - **Loop** inductance
  - **Frequency dependence** due to skin effect
- Per-unit-length resistance *R*
  - **Strong frequency dependence** due to skin effect



Time-Domain Solution of Telegrapher's Equations

Neglecting losses for simplicity:

$$\left.\begin{aligned}\frac{\partial v(z,t)}{\partial z} &= -L\frac{\partial i(z,t)}{\partial t}\\ \frac{\partial i(z,t)}{\partial z} &= -C\frac{\partial v(z,t)}{\partial t}\end{aligned}\right\} \Rightarrow \boxed{\frac{\partial^2 v}{\partial z^2} - \frac{1}{v_p^2}\frac{\partial^2 v}{\partial t^2} = 0}$$

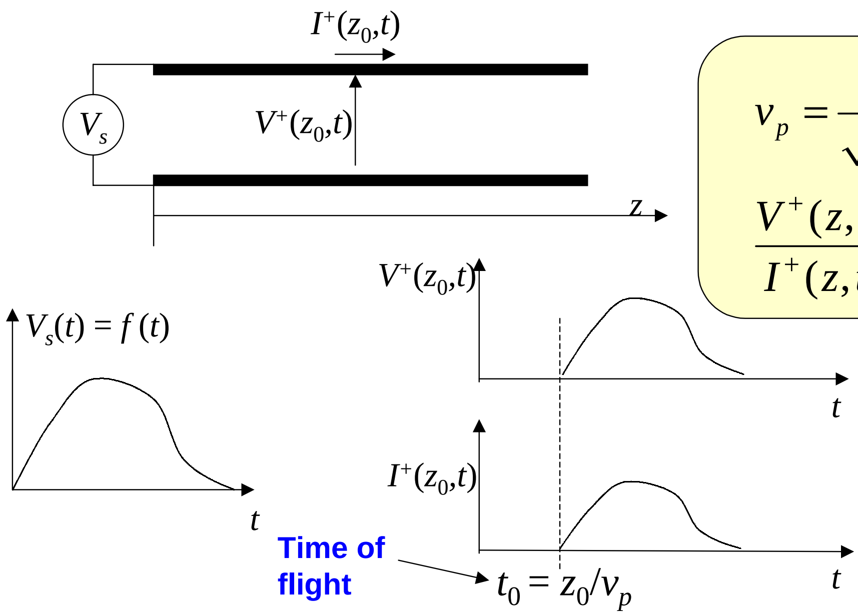
where $v_p = 1/\sqrt{LC}$ is the wave velocity on the line.

General solution: $v(z,t) = \underbrace{f^+(z - v_p t)}_{\text{forward wave}} + \underbrace{f^-(z + v_p t)}_{\text{backward wave}}$

Current wave: $i(z,t) = \underbrace{\frac{1}{Z_0} f^+(z - v_p t)}_{\text{forward wave}} - \underbrace{\frac{1}{Z_0} f^-(z + v_p t)}_{\text{backward wave}}$

where $Z_0 = \sqrt{\frac{L}{C}}$ is the **characteristic impedance** of the line.

A voltage signal *f (t)* launched on a lossless line propagates unaltered with speed v_p dependent on the transmission-line properties.

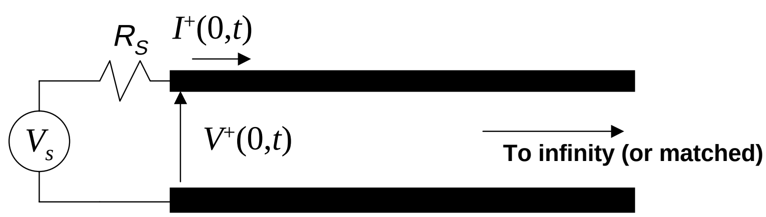


$$v_p = \frac{1}{\sqrt{LC}}$$

$$\frac{V^+(z,t)}{I^+(z,t)} \triangleq Z_0 = \sqrt{\frac{L}{C}}$$

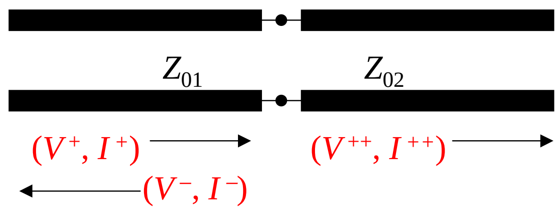
Characteristic Impedance

The characteristic impedance dictates the amplitude of the voltage waveform launched on the line



$$\left.\begin{array}{l} V_S(t) = V^+(0,t) + I^+(0,t)R_S \\ V^+(0,t) = Z_0 I^+(0,t) \end{array}\right\} \Rightarrow V^+(0,t) = V_S(t)\frac{Z_0}{Z_0 + R_S}$$

Discontinuities in the characteristic impedance of a transmission line give rise to reflections



At the junction it is:

$$V_1 = V^+ + V^- = V_2 = V^{++}$$

$$I_1 = \frac{1}{Z_{01}}\left(V^+ - V^-\right) = I_2 = \frac{V^{++}}{Z_{02}}$$

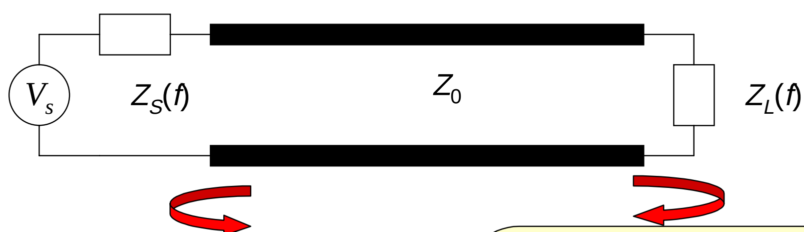
$$\left.\begin{aligned} V^- &= \Gamma V^+ \\ I^- &= -\frac{V^-}{Z_{01}} \end{aligned}\right\} \quad \text{and} \quad \left.\begin{aligned} V^{++} &= T V^+ \\ I^{++} &= \frac{V^{++}}{Z_{02}} \end{aligned}\right\}$$

Reflection Coefficient: $\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}}$

Transmission Coefficient: $T = \frac{2Z_{02}}{Z_{02} + Z_{01}}$

Maintaining a fairly constant value of the characteristic impedance along an interconnect path is essential for reflection suppression.

Source and load impedances impact transmission line performance of the interconnect



Source reflection coefficient

$$\Gamma_S(f) = \frac{Z_S(f) - Z_0}{Z_S(f) + Z_0}$$

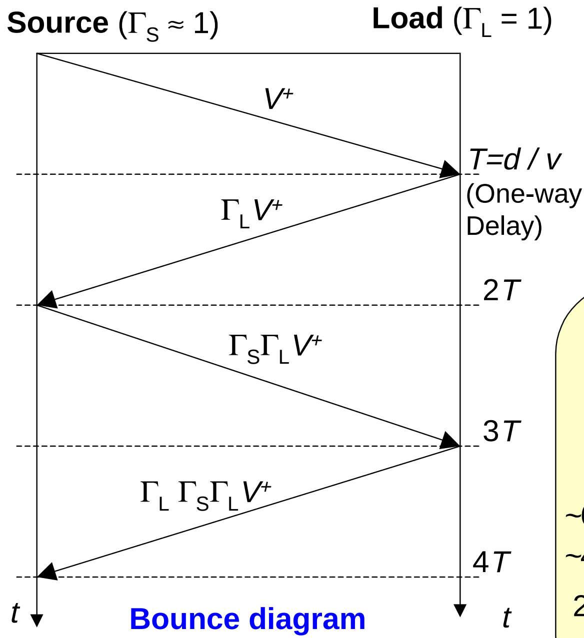
Load reflection coefficient:

$$\Gamma_L(f) = \frac{Z_L(f) - Z_0}{Z_L(f) + Z_0}$$

Load transmission coefficient:

$$T_L(f) = 1 + \Gamma_L(f) = \frac{2Z_L(f)}{Z_L(f) + Z_0}$$

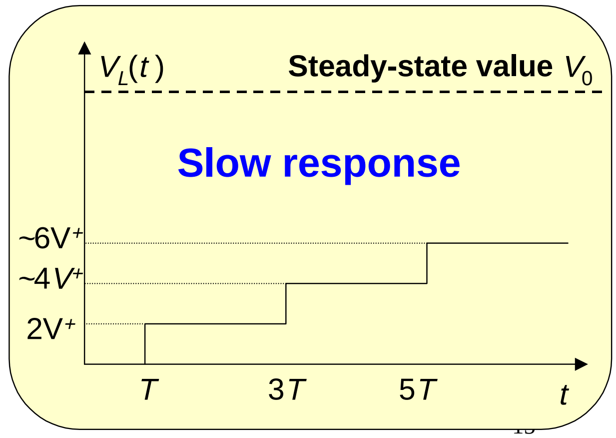
Example: Unterminated interconnect ($Z_L=\infty$) driven by high source impedance driver with $Z_S >> Z_0$ (e.g. unbuffered CMOS)



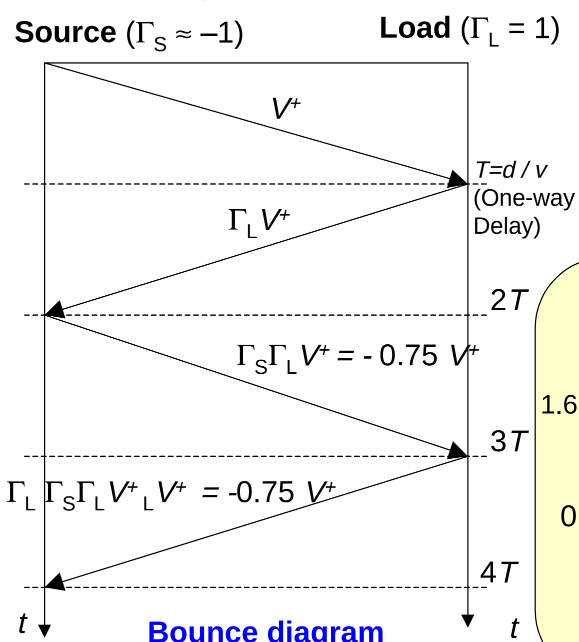
Excitation: Step Pulse of amplitude V_0

$$V^+ = V_0 \frac{Z_0}{Z_S + Z_0} << V_0, \quad \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \approx 1$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 1, \quad T_L = 2$$



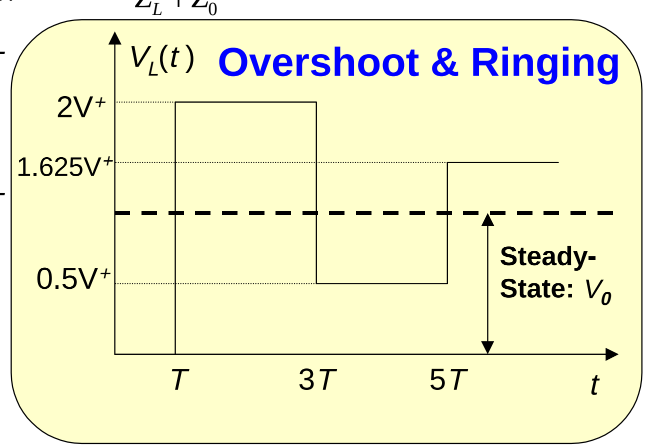
Example: Unterminated interconnect ($Z_L=\infty$) driven by low source impedance driver with $Z_S<Z_0$ (e.g. ECL or strong TTL)



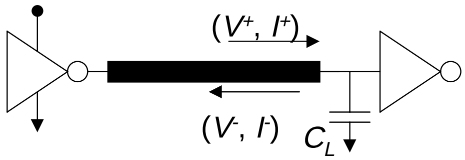
Excitation: Step Pulse of amplitude V_0; $Z_0 = 7Z_S$

$$V^+ = V_0 \frac{Z_0}{Z_S + Z_0} \approx \frac{7}{8} V_0, \quad \Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = -0.75$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 1, \quad T_L = 2$$



A capacitor C_L represents the load at the gate input of the receiver. Its presence adds delay.



Interconnect delay = T

$$V_L = V^+ + V^-$$

$$I_L = \frac{1}{Z_0}\left(V^+ - V^-\right)$$

$$I_L = C_L \frac{dV_L}{dt},\ \ V_L(t = T) = 0.$$

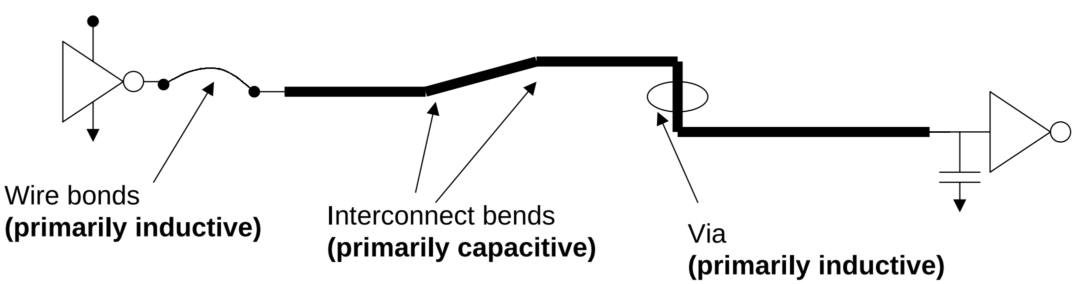
$$V_L(t) = V^+\left(1 - \exp(-(t-T)/\tau)\right), \quad t > T$$
where $\tau = Z_0 C$.

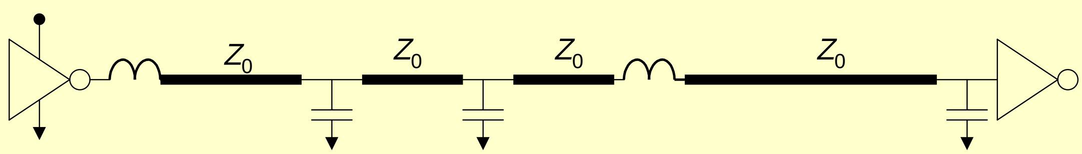
Let T_d be the time at which $V_L(t = T_d) = 0.9V^+$.

$T_d = T + 2.3\tau \Rightarrow$

Extra delay due to the capacitor is $2.3Z_0C$

Delay is introduced by all capacitive and inductive discontinuities present in a signal path

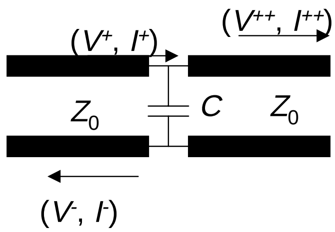




Equivalent circuit for SPICE-based transient simulation

The delay due to a capacitive o an inductive discontinuity depends on the values *C* or *L* and Z_0

Capacitive Discontinuity



$$V^{++}(t) = V^{+}\left(1 - \exp(-t/\tau_C)\right)$$

$$\text{where } \tau_C = \frac{CZ_0}{2}$$

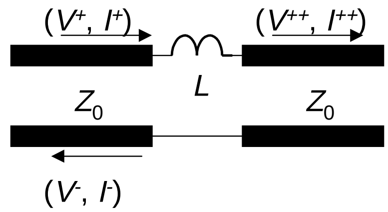
$V^{++}(t)$ reaches $0.9V^{+}$ at $t = 2.3\tau_C$

Hence, the capacitor adds a delay of

$T_d = 1.15CZ_0$.

(C=1 pF, $Z_0 = 50$ Ohm; $T_d = 57.5$ ps)

Inductive Discontinuity



$$V^{++}(t) = V^{+}\left(1 - \exp(-t/\tau_L)\right)$$

$$\text{where } \tau_L = \frac{L}{2Z_0}$$

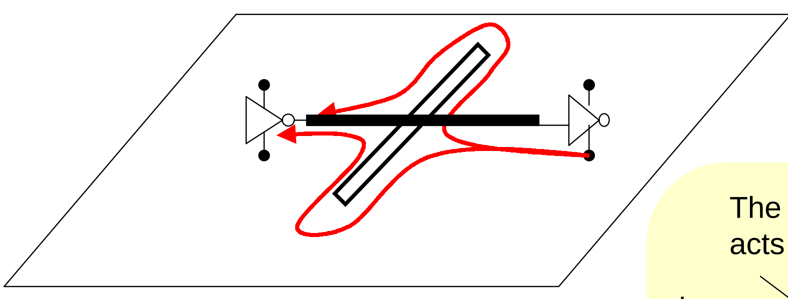
$V^{++}(t)$ reaches $0.9V^{+}$ at $t = 2.3\tau_L$

Hence, the inductor adds a delay of

$T_d = 1.15L/Z_0$

($L = 2.5$ nH, $Z_0 = 50$ Ohm; $T_d = 57.5$ ps)

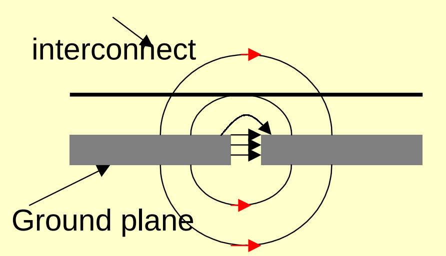
Slots in ground planes increase interconnect delay and enhance noise generation and interference



The return current in the ground plane flows around the slot. Hence,

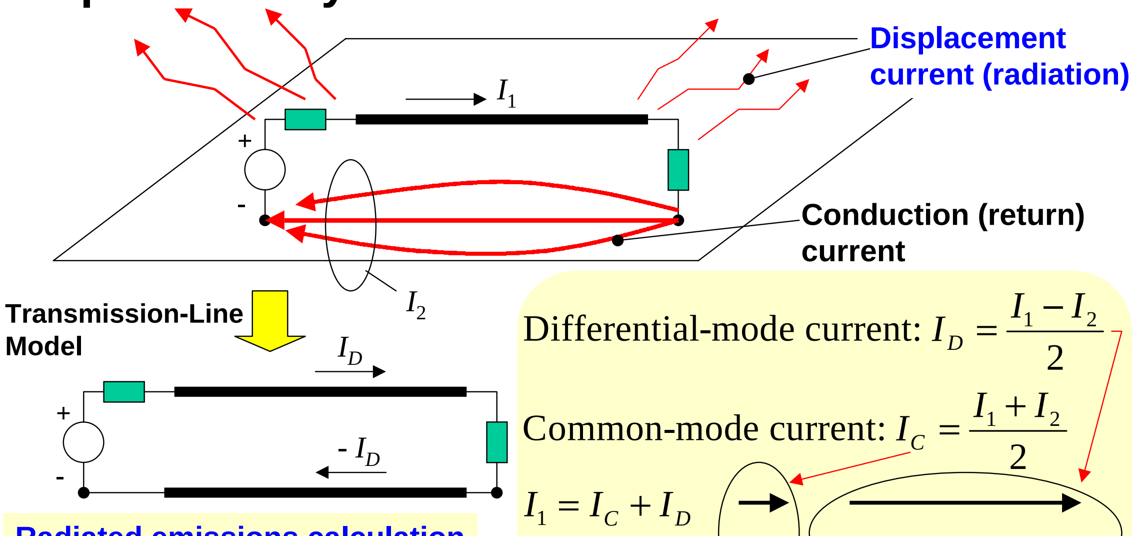
- **Extra *L* ⇒ extra delay**
- **Unbalanced currents lead to enhanced emissions**
- **Interference (crosstalk) with other wires beyond immediate neighbors**

The slot in the ground plane acts as a **slot antenna.**



Interference occurs on either side of the plane

Transmission line models of interconnects predict only "differential-mode" currents



Differential-mode current: $I_D = \dfrac{I_1 - I_2}{2}$

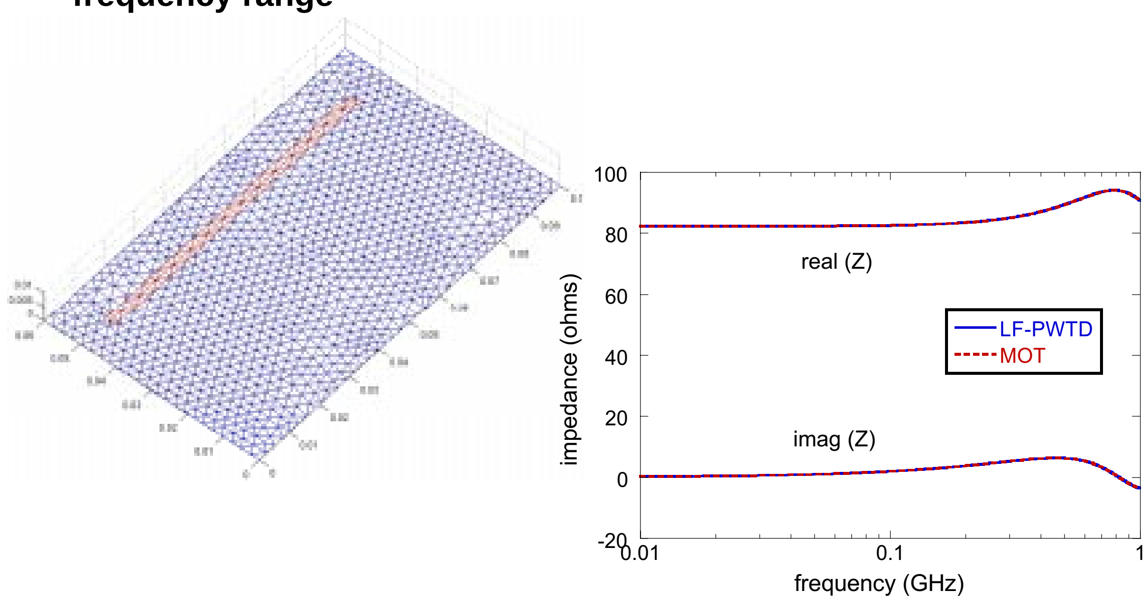
Common-mode current: $I_C = \dfrac{I_1 + I_2}{2}$

$I_1 = I_C + I_D$

$I_2 = I_C - I_D$

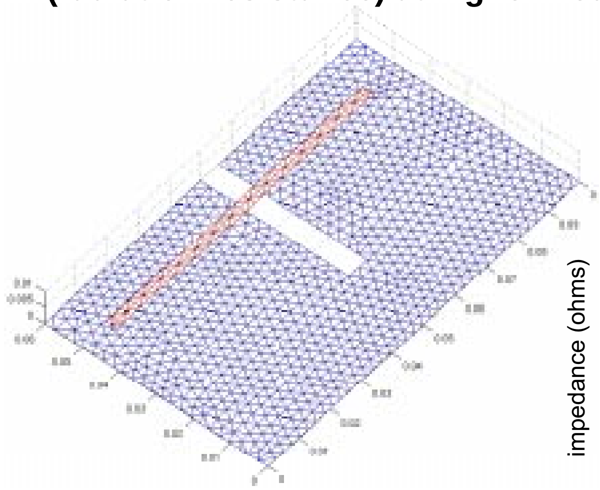
Radiated emissions calculation can be grossly incorrect if the common-mode current is not taken into account

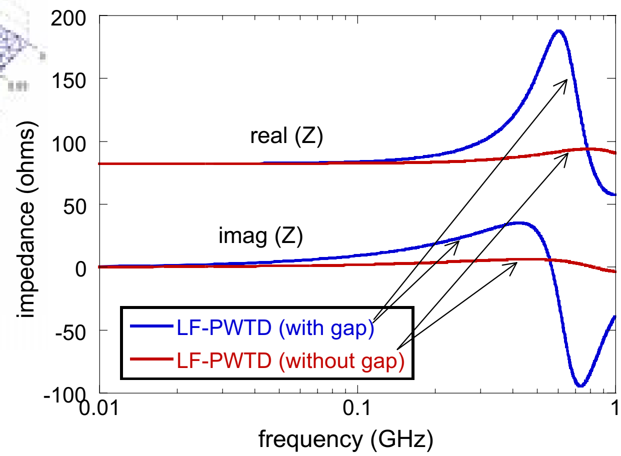
The input impedance of a match-terminated interconnect with a continuous return path remains essentially contact over a broad frequency range



Plots generated using UIUC's fast time-domain solvers (Prof. E. Michielssen)

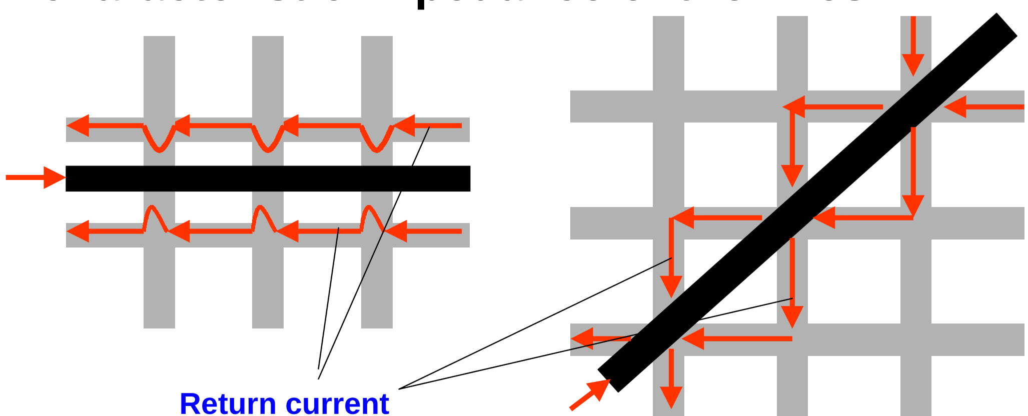
The disruption of the return path caused by the slot manifests itself as an added inductance at lower frequencies and radiated emissions (radiation resistance) at higher frequencies





Plots generated using UIUC's fast time-domain EM solvers (Prof. E. Michielssen)

Mesh (Grid) Planes in PCBs increase the characteristic impedance of the lines

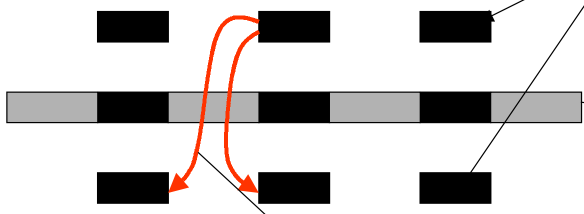


Per-unit-length inductance, *L*, increases.

Per-unit-length capacitance, *C*, decreases.

$$Z_0 = \sqrt{\frac{L}{C}}$$ increases

In the case of mesh (grid) planes, high-speed lines should be routed right above the plane metallization



Advantages

-Better impedance control

-Reduced cross-plane interference

Lossy Transmission Lines

- **Ohmic loss in the metallization**
  - Frequency-dependent *R* and *L* (skin effect)
- **Insulating substrate loss**
  - Frequency-dependent *G*
- **Semiconductor substrates**
  - Frequency-dependent *R* and *L*
  - Frequency-dependent *G* and *C*

Frequency-Domain Solution of Telegrapher's Equations

In the frequency domain, interconnect loss can be accounted for easily.

$$\left.\begin{aligned} -\frac{dV(z,\omega)}{dz} &= [R(\omega)+j\omega L(\omega)]\,I(z,\omega) \\ -\frac{dI(z,\omega)}{dz} &= [G(\omega)+j\omega C(\omega)]\,V(z,\omega) \end{aligned}\right\} \Rightarrow \frac{d^2V(z,\omega)}{dz^2} - Z(\omega)Y(\omega)V(z,\omega) = 0$$

where $Z(\omega) = R(\omega) + j\omega L(\omega)$ is the per-unit-length impedance of the line and $Y(\omega) = G(\omega) + j\omega C(\omega)$ is the per-unit-length admittance of the line.

General solution :

$$\begin{cases} V(z,\omega) = V^+(\omega)\exp(-\gamma z) + V^-(\omega)\exp(\gamma z) \\ I(z,\omega) = \dfrac{1}{Z_0(\omega)}\left[V^+(\omega)\exp(-\gamma z) - V^-(\omega)\exp(\gamma z)\right] \end{cases}$$

$\gamma(\omega) = \sqrt{[R(\omega)+j\omega L(\omega)][G(\omega)+j\omega C(\omega)]}$ is the **complex propagation constant**,

and $Z_0(\omega) = \sqrt{\dfrac{R(\omega)+j\omega L(\omega)}{G(\omega)+j\omega C(\omega)}}$ is the **characteristic impedance**.

The characteristic impedance of a lossy line is a complex number!

When $G \approx 0$ it is the per-unit-length ohmic loss in the wires that dominates the loss; hence,

$$Z_0(\omega) = \sqrt{\frac{R(\omega)+jL(\omega)}{j\omega C}} = \sqrt{\frac{L(\omega)}{C}}\sqrt{1 - j\frac{R(\omega)}{\omega L(\omega)}}$$

For the interconnect structures of interest , L is in the order of nH/cm; hence, for $f <$ a few tens of MHz, $\omega L << R$ (especially for thin-film wire).

Thus, for low frequencies: $Z_0(\omega) \approx \dfrac{1-j}{\sqrt{2}}\sqrt{\dfrac{R(\omega)}{\omega C}}$

Notice that the real and imaginary parts are of the same magnitude.

On the other hand, for high frequencies such that $R << \omega L$,

$$Z_0(\omega) = \sqrt{\frac{L}{C}}\left(1 - j\frac{1}{2}\frac{R(\omega)}{\omega L(\omega)}\right)$$

Notice that, since at high frequencies $R(\omega) \propto \sqrt{\omega}$, the characteristic impedance is predominantly real.

The presence of loss is responsible for signal attenuation and distortion

The propagation constant becomes frequency dependent:

$$\gamma(\omega)=\sqrt{[R(\omega)+j\omega L(\omega)][G(\omega)+j\omega C]}=\alpha(\omega)+j\beta(\omega).$$

$\alpha(\omega)$ is the **attenuation constant;** $\beta(\omega)$ is the **phase constant**.

$$V^{+}(\omega,z)=|V_0^{+}|\underbrace{\exp(-\alpha(\omega)z)}_{\text{attenuation}}\underbrace{\exp(-j\beta(\omega)z)}_{\text{phase shift}}$$

The characteristic impedance and the phase velocity are frequency dependent:

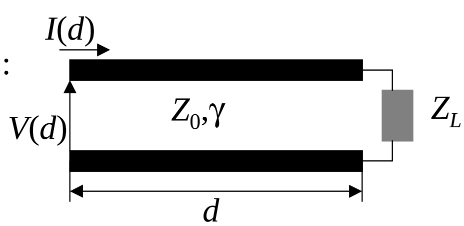
$$Z_0(\omega)=\sqrt{\frac{R(\omega)+j\omega L(\omega)}{G(\omega)+j\omega C}},\quad v_p(\omega)=\frac{\omega}{\beta(\omega)}$$

Different frequencies in the spectrum of a pulse propagate at different speeds and suffer different attenuation.
This results in pulse distortion often referred to as **dispersion**

Input Impedance of a Transmission Line

(or, how long wires "transform" load impedances)

$$Z_{in}(d)=\frac{V(d)}{I(d)}=Z_0\frac{Z_L+Z_0\tanh\gamma d}{Z_0+Z_L\tanh\gamma d}$$



Neglecting losses, $\gamma=j\beta$, Z_0 is real, and it is:

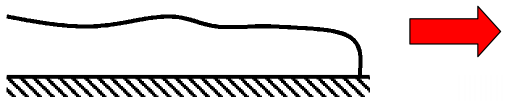
$$Z_{in}(d)=Z_0\frac{Z_L+jZ_0\tan\beta d}{Z_0+jZ_L\tan\beta d}$$

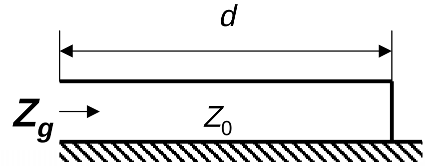
– Periodic function with period $\lambda/2$

– $\max(Z_{in})=Z_0\dfrac{1+\left|\dfrac{Z_L-Z_0}{Z_L+Z_0}\right|}{1-\left|\dfrac{Z_L-Z_0}{Z_L+Z_0}\right|}$; $\min(Z_{in})=Z_0\dfrac{1-\left|\dfrac{Z_L-Z_0}{Z_L+Z_0}\right|}{1+\left|\dfrac{Z_L-Z_0}{Z_L+Z_0}\right|}$

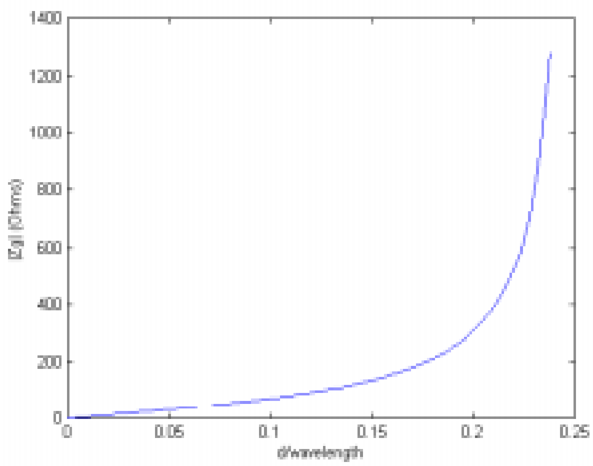
- Matched Load: $Z_L=Z_0\Rightarrow Z_{in}(d)=Z_0$
- Shorted Line: $Z_L=0\Rightarrow Z_{in}(d)=jZ_0\tan\beta d$
  - **A shorted line of length equal to an odd multiple of $\lambda/4$ has infinite input impedance and thus appears as an open circuit.**

Grounding wires running some distance along a ground plane or chassis exhibit transmission-line behavior at RF frequencies.

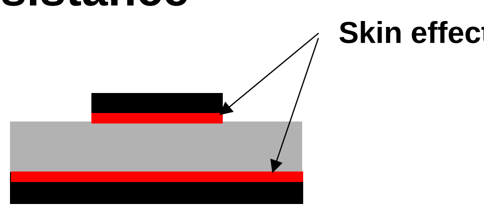




- Safety earth is not an RF ground.
- At high frequencies, the claim that "everything is connected to earth" through safety earth is meaningless
- At high frequencies, "single point ground" is meaningless



Skin-Effect Resistance



At frequencies such that the skin depth is larger or comparable with the conductor thickness, the current distributes uniformly over the conductor cross section.

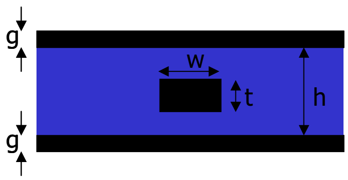
At high frequencies, where the skin depth is smaller than the conductor thickness, **current crowding** around the perimeter occurs.

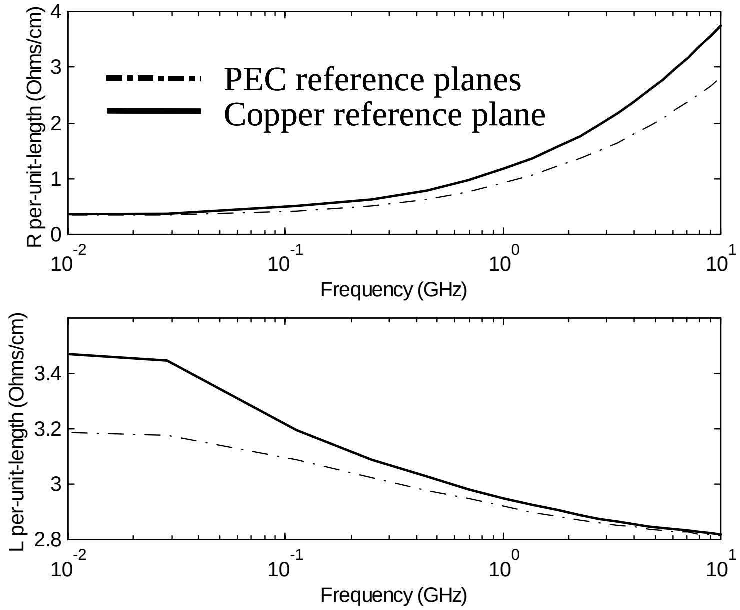
$$\text{Skin depth: } \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

- At $f = 1$ GHz, for aluminum with conductivity $\sigma = 4 \times 10^7$ S/m and permeability $\mu = 4\pi \times 10^{-7}$ H/m, the skin depth is 2.5 μm.
- **For high enough frequencies, the p.u.l. resistance increases as $\sqrt{f}$**

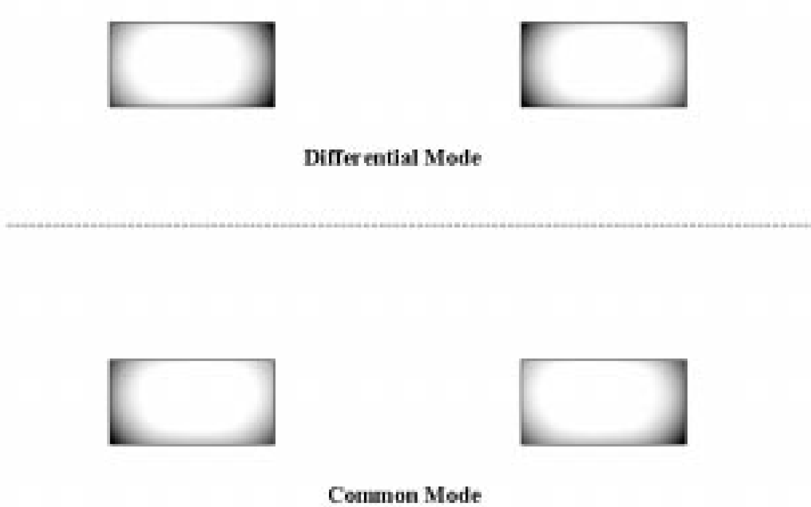
The contribution of the return path to interconnect resistance may need to be taken into account

Frequency dependence of the p.u.l. resistance (top) and inductance (bottom) of the single stripline configuration with w=50 µm, t=10 µm, g=10 µm, and h=100 µm.

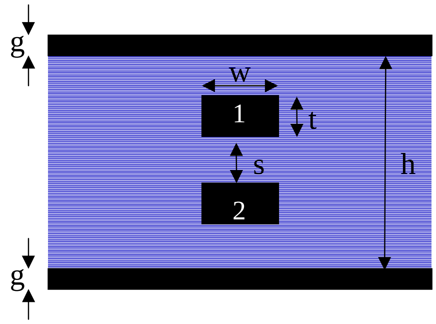


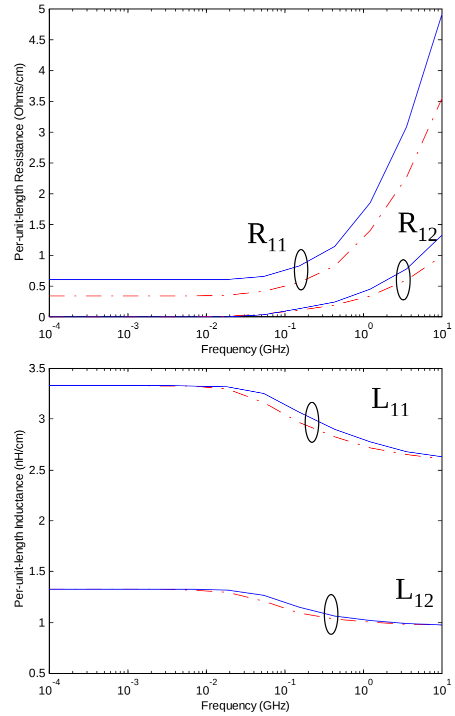


Extraction of the frequency-dependent p.u.l. interconnect resistance must take into account the presence of adjacent conductors (proximity effect)



The per-unit-length resistance matrix has non-zero off-diagonal elements. Taking these off-diagonal elements into account is important, especially for the tightly coupled wires





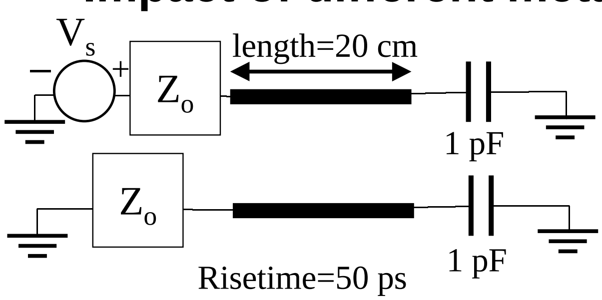
w= 50 µm, t=10 µm, s= 30 µm

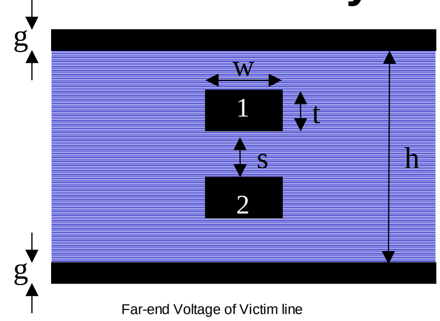
h= 110 µm, g= 10 µm, ε_r=4

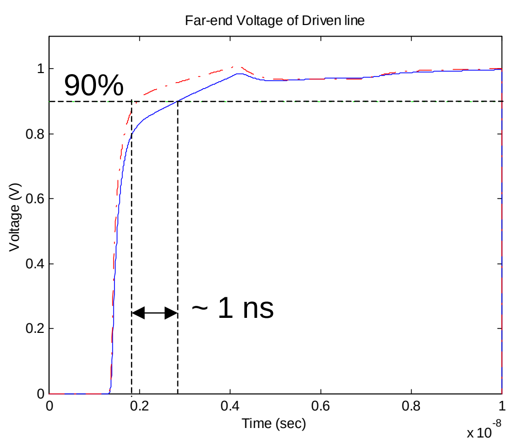
Aluminum: σ=3.3E7 S/m

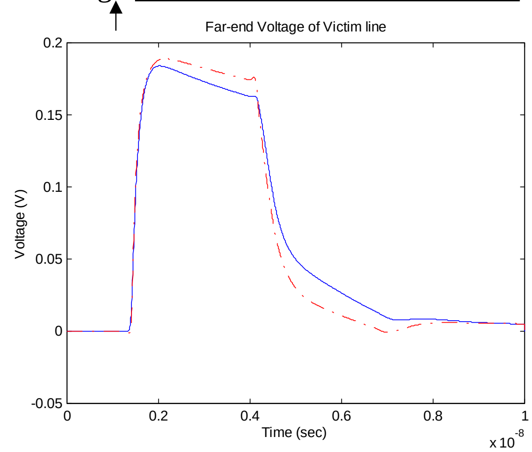
Copper: σ =5.8E7 S/m

Impact of different metallization on delay

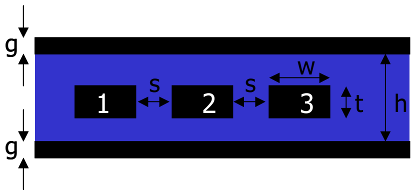




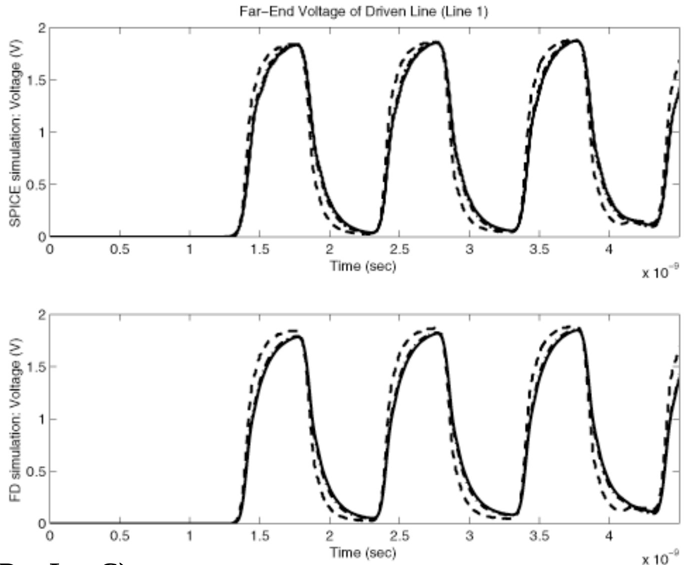




Impact of frequency-dependent loss on interconnect transient response

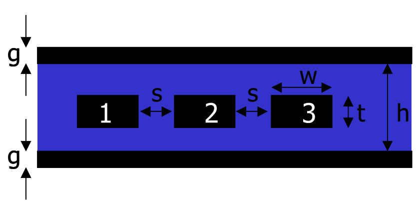


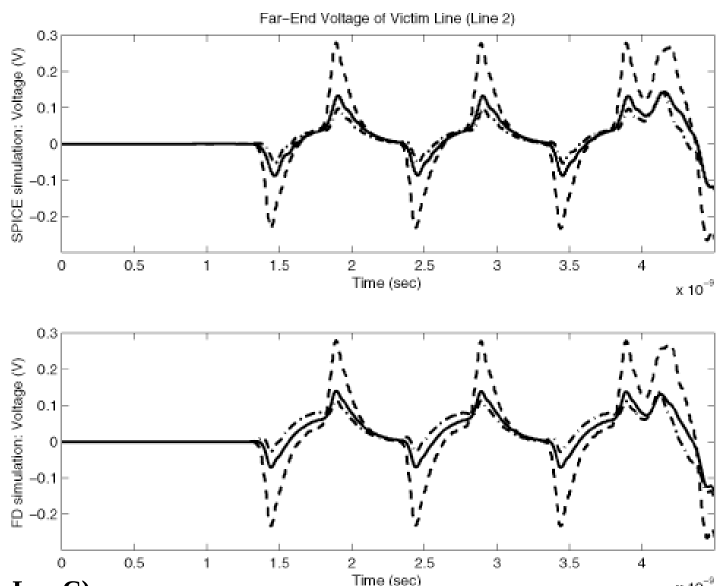
Cross-section of a stripline geometry.
s=50 µm, w=50 µm,
t=10 µm, g=10 µm,
h=200 µm and ε_r=4.
Copper metallization.



Constant Model (R_{dc}, L_8 , C)
Frequency dependent model: PEC reference planes (R(f), L(f), C)
Frequency dependent model: Copper reference plane (R(f), L(f), C)

The effect of frequency-dependent loss is particularly apparent in the cross-talk levels





Constant Model (R_{dc}, L_8 , C)
Frequency dependent model: PEC reference planes (R(f), L(f), C)
Frequency dependent model: Copper reference plane (R(f), L(f), C)

Insulating Substrate Loss

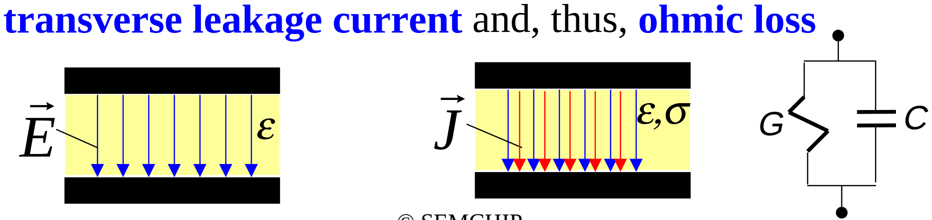
- Characterized in terms of the substrate material conductivity or **loss tangent**

$$\varepsilon(\omega) = \varepsilon'(\omega) - j\varepsilon''(\omega) = \varepsilon'(\omega)\left(1 - j\frac{\varepsilon''(\omega)}{\varepsilon'(\omega)}\right) = \varepsilon'(\omega)\left(1 - j\tan\delta(\omega)\right)$$

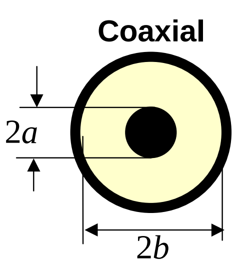
$$\sigma(\omega) + j\omega\varepsilon'(\omega) = j\omega\left\{\varepsilon'(\omega)\left(1 - j\frac{\sigma(\omega)}{\omega\varepsilon'(\omega)}\right)\right\}$$

$$\tan\delta(\omega) = \frac{\varepsilon''(\omega)}{\varepsilon'(\omega)} = \frac{\sigma(\omega)}{\omega\varepsilon'(\omega)}$$

- Transverse electric field between conductors results in a **transverse leakage current** and, thus, **ohmic loss**



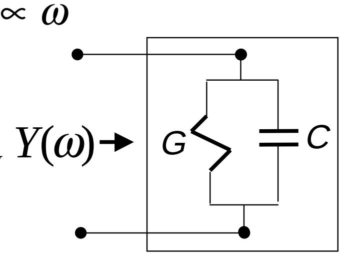
The assumption of constant loss tangent leads to physically inconsistent models for *G*



$$C = \frac{2\pi\varepsilon}{\ln(b/a)}, \quad G = \frac{2\pi\sigma}{\ln(b/a)} \Rightarrow$$

$$\frac{G}{C} = \frac{\sigma}{\varepsilon} \Rightarrow \frac{G}{C} = \omega\tan\delta$$

- Assuming $\tan\delta$ is constant yields $G(\omega) \propto \omega$
  - **Such a behavior violates causality!**
  - For a causal circuit
    $\mathrm{Re}\{Y(\omega)\}$ is an **even function** of frequency
    $\mathrm{Im}\{Y(\omega)\}$ is an odd function of frequency



Simply assuming the loss tangent to remain constant over a broad (multi-GHz) frequency range leads to a non-physical behavior of *G*(*ω*)

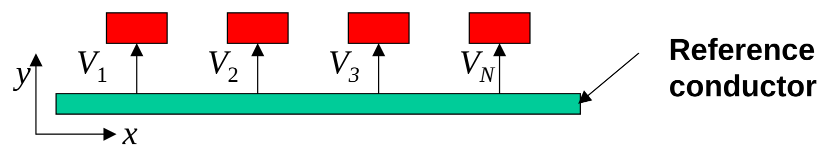
- A **physically correct model** needs to start with a physically-correct description of the frequency dependence of the complex permittivity.
  - Use measured data for the complex permittivity to synthesize a **Debye model** for it
  - Use the synthesized Debye model for the extraction of $C(\omega)$ and $G(\omega)$

$$\varepsilon(\omega) = \varepsilon'(\omega) - j\varepsilon''(\omega) = \varepsilon_\infty + \sum_{k=1}^{K} \frac{\varepsilon_k}{1 + j\omega\tau_k} \Rightarrow$$

$$\tan\delta(\omega) = \frac{\varepsilon''(\omega)}{\varepsilon'(\omega)} = \frac{\displaystyle\sum_{k=1}^{K} \frac{\omega\varepsilon_k\tau_k}{1+\omega^2\tau_k^2}}{\displaystyle\varepsilon_\infty + \sum_{k=1}^{K} \frac{\varepsilon_k}{1+\omega^2\tau_k^2}}$$

$G(\omega) \propto \omega \tan\delta \Rightarrow$ **even function of frequency**

Capacitive and Inductive Crosstalk in Short Interconnects



$$-\frac{d}{dz}\begin{Bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{Bmatrix} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{bmatrix} \begin{Bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{Bmatrix} + j\omega \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1N} \\ L_{21} & L_{22} & \cdots & L_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ L_{N1} & L_{N2} & \cdots & L_{NN} \end{bmatrix} \begin{Bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{Bmatrix}$$

$$-\frac{d}{dz}\begin{Bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{Bmatrix} = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{Bmatrix} + j\omega \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ C_{21} & C_{22} & \cdots & C_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ C_{N1} & C_{N2} & \cdots & C_{NN} \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{Bmatrix}$$

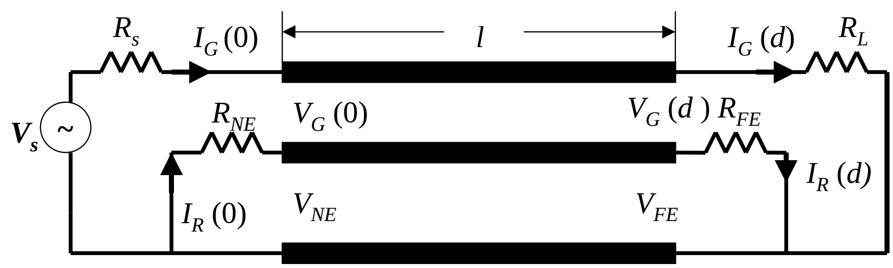
Crosstalk in Coupled Lines

- For interconnects with more than two (active) conductors, crosstalk analysis is most effectively performed in terms of a circuit simulator that can support MTL models (*).
  - **Most common (and computationally efficient) SPICE equivalent circuits for MTL assume lossless transmission lines.**
  - **Models for MTLs with losses (including frequency-dependent losses associated with skin effect) are available also. They are essential for accurate analysis of interconnect-induced delay, dispersion, and crosstalk at the board level for signals of GHz bandwidths.**
  - **It is assumed that the interconnect structure is uniform enough for its description in terms of per-unit-length *L,C,R,* and *G* matrices to make sense.**

(*) V.K. Tripathi and J.B. Rettig, "A SPICE Model for Multiple Coupled Microstrips and Other Transmission Lines," *IEEE Trans. Microwave Theory Tech.,* vol. 33(12), pp. 1513-1518, Dec. 1985.

For the case of a three-conductor, lossless line in homogeneous dielectric, with resistive terminations, an exact solution is possible.

- Exact solutions are useful because:
  - **they help provide insight into the crosstalk mechanism;**
  - **they can be used to validate computer-based simulations.**
- The following results were first published by C.R. Paul (C.R. Paul, "Solution of transmission line equations for three-conductor lines in homogeneous media," *IEEE Trans. On Electromagnetic Compatibility,* vol. 20, pp. 216-222, 1978.



Exact solution for crosstalk in a lossless, three-conductor line with resistive terminations

Per unit length parameters: $\mathbf{L}=\begin{bmatrix} L_G & M \\ M & L_R \end{bmatrix}, \quad \mathbf{C}=\begin{bmatrix} C_G & -C_M \\ -C_M & C_R \end{bmatrix}$

Near-end and far-end crosstalk voltages:

$$V_{NE}=\frac{S}{D}\left[\frac{R_{NE}}{R_{NE}+R_{FE}}j\omega Ml\left(C+\frac{j2\pi l/\lambda}{\sqrt{1-k^2}}\alpha_{LG}S\right)I_{G_{DC}}\right]+$$

$$\frac{S}{D}\left[\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}j\omega C_M l\left(C+\frac{j2\pi l/\lambda}{\sqrt{1-k^2}}\frac{1}{\alpha_{LG}}S\right)V_{G_{DC}}\right]$$

$$V_{FE}=\frac{S}{D}\left[-\frac{R_{FE}}{R_{NE}+R_{FE}}j\omega MlI_{G_{DC}}+\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}}j\omega C_M lV_{G_{DC}}\right]$$

where, $C=\cos\beta l,\ S=\frac{\sin\beta l}{\beta l}$, $k=\frac{M}{\sqrt{L_G L_R}}=\frac{C_M}{\sqrt{C_G C_R}}\le 1$, and

Inductive & Capacitive Coupling Coefficients

Exact solution for crosstalk in a lossless, three-conductor line with resistive terminations

$$D=C^2-S^2\omega^2\tau_G\tau_R\left[1-k^2\frac{(1-\alpha_{SG}\alpha_{LR})(1-\alpha_{LG}\alpha_{SR})}{(1+\alpha_{SR}\alpha_{LR})(1+\alpha_{SG}\alpha_{LG})}\right]+j\omega CS(\tau_G+\tau_R);$$

$$\alpha_{SG}=\frac{R_S}{Z_{CG}},\quad \alpha_{LG}=\frac{R_L}{Z_{CG}},\quad \alpha_{SR}=\frac{R_{NE}}{Z_{CR}},\quad \alpha_{LR}=\frac{R_{FE}}{Z_{CR}};$$

$Z_{CG}=\sqrt{L_G/C_G}$, $Z_{CR}=\sqrt{L_R/C_R}$ are the characteristic impedances of each line in the presence of the other one;

$$\tau_G=\frac{L_G l}{R_S+R_L}+C_G l\frac{R_S R_L}{R_S+R_L},\quad \tau_R=\frac{L_R l}{R_{NE}+R_{FE}}+C_R l\frac{R_{NE}R_{FE}}{R_{NE}+R_{FE}},$$

are the time constants of the coupled lines;

$V_{G_{DC}}=\frac{R_L}{R_S+R_L}V_S,\quad I_{G_{DC}}=\frac{V_S}{R_S+R_L}$, are the voltage and current of the generator circuit under dc excitation (no coupling to the receptor circuit).

Under the assumptions of electrically short lines, and weak coupling, the crosstalk equations simplify considerably

- A line is said to be **electrically short** if its length is a small fraction of the wavelength at the highest frequency of interest.
  Package interconnects fall in this category
- Two lines are said to be **weakly coupled** if the coupling coefficient, *k* , is sufficiently smaller than 1.

Under these assumptions the equations for the near-end and far-end crosstalk voltages become:

$$V_{NE} = \frac{1}{D}\left(\frac{R_{NE}}{(R_{NE}+R_{FE})(R_S+R_L)} j\omega Ml + \frac{R_{NE}R_{FE}R_L}{(R_{NE}+R_{FE})(R_S+R_L)} j\omega C_M l \right)$$

$$V_{FE} = \frac{1}{D}\left(-\frac{R_{FE}}{(R_{NE}+R_{FE})(R_S+R_L)} j\omega Ml + \frac{R_{NE}R_{FE}R_L}{(R_{NE}+R_{FE})(R_S+R_L)} j\omega C_M l \right)$$

For weakly coupled, electrically short wires, the crosstalk is a linear combination of contributions due to the mutual inductance between the lines (inductive coupling) and the mutual capacitance between the lines (capacitive coupling).

$$V_{NE} = V_{NE}^{IND} + V_{NE}^{CAP} = \frac{j\omega}{D}\frac{R_{NE}}{(R_{NE}+R_{FE})(R_S+R_L)}\left(Ml + (R_{FE}R_L)C_M l\right)$$

$$V_{FE} = V_{FE}^{IND} + V_{FE}^{CAP} = \frac{j\omega}{D}\frac{R_{NE}}{(R_{NE}+R_{FE})(R_S+R_L)}\left(-Ml + (R_{NE}R_L)C_M l\right)$$

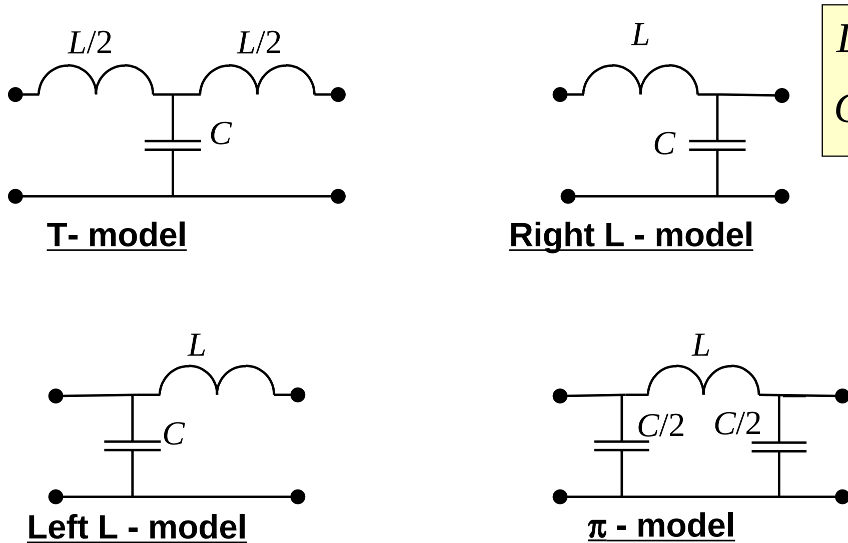
Notice that:

- The higher the frequency the larger the crosstalk
- Inductive coupling dominates for low-impedance loads
- Capacitive coupling dominates for high-impedance loads

Lumped versus Distributed Modeling

- When the interconnect length is much smaller than the wavelength of interest, **lumped models** provide sufficient accuracy and can be used
  - **Typical case for package interconnects at RF frequencies**
  - **Inaccurate for interconnects at the MCM and PCB level**
- What does ***"interconnect length is much smaller than the wavelength of interest"*** really mean?
  - Typical rule of thumb: $d < \lambda/10$
  - …but one can take a closer look at this rule of thumb as shown next

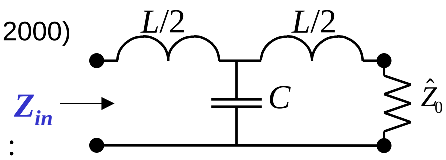
There are four possible implementations of lumped models for a two-conductor interconnect of length *d*



$$L = L_{p.u.l.}d$$
$$C = C_{p.u.l.}d$$

The accuracy of a lumped model can be examined by considering the input impedance obtained when the model is terminated at the characteristic impedance of the line

(see B. Young, ***Digital Signal Integrity***, Prentice Hall, 2000)



For the model to exhibit "transmission line" behavior, its input impedance should equal the load impedance $\hat{Z}_0$:

$$Z_{in} = j\omega L/2 + \frac{1}{j\omega C + \frac{1}{\hat{Z}_0 + j\omega L/2}} = \hat{Z}_0 \Rightarrow \boxed{\hat{Z}_0 = \sqrt{\frac{L}{C}}\sqrt{1-\left(\frac{\omega}{\omega_T}\right)^2}, \text{ where } \omega_T = \frac{2}{\sqrt{LC}}}$$

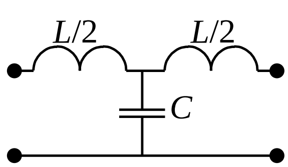
- For $\omega \ll \omega_T$, $\hat{Z}_0 \approx Z_0$
- A bandwidth of validity of the T-model can be obtained by finding $\omega_{\max}$ such that $Z_0 - \hat{Z}_0 \le aZ_0$ for $\omega \le \omega_{\max}$.

$$\boxed{\text{For } a = 0.05,\ \omega_{\max} = \frac{0.62}{\sqrt{LC}} = \frac{0.62}{d\sqrt{L_{p.u.l}C_{p.u.l.}}}.}$$

Application: $d = 2$ cm, $L_{p.u.l.} = 4$ nH/cm, $C_{p.u.l} = 1$ pF/cm; $\omega_{\max} = 4.9$ GHz

5% accuracy bandwidth for the four lumped models

T- model



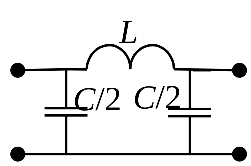
$$\hat{Z}_0 = \sqrt{\frac{L}{C}}\sqrt{1-\left(\frac{\omega\sqrt{LC}}{2}\right)^2}$$

$$\omega_{\max} = \frac{0.62}{\sqrt{LC}}$$

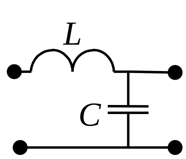
π - model

$$\hat{Z}_0 = \sqrt{\frac{L}{C}}\left[1-\left(\frac{\omega\sqrt{LC}}{2}\right)^2\right]^{-1/2}$$

$$\omega_{\max} = \frac{0.62}{\sqrt{LC}}$$



Right L - model



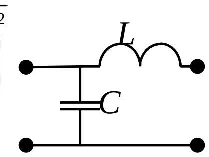
$$\hat{Z}_0 = j\omega L/2 + \sqrt{\frac{L}{C}}\sqrt{1-\left(\frac{\omega\sqrt{LC}}{2}\right)^2}$$

$$\omega_{\max} = \frac{0.1}{\sqrt{LC}}$$

Left L - model

$$\hat{Z}_0 = -j\omega L/2 + \sqrt{\frac{L}{C}}\sqrt{1-\left(\frac{\omega\sqrt{LC}}{2}\right)^2}$$

$$\omega_{\max} = \frac{0.1}{\sqrt{LC}}$$



The number of segments is dictated by the maximum frequency of interest that must be represented accurately in the simulation

- The effective bandwidth criteria described earlier can be used to select the segment size.

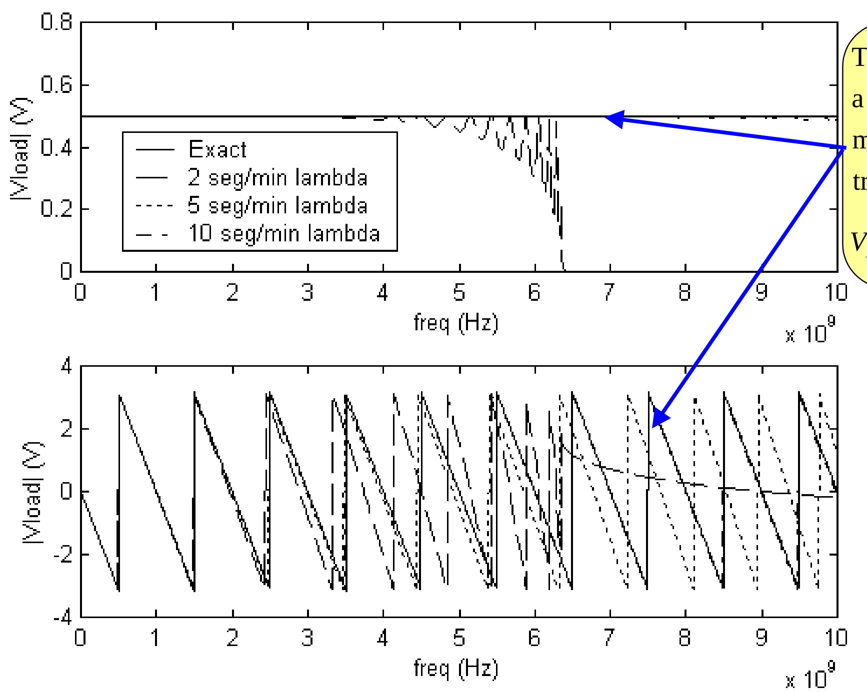
Example : Interconnect lead with total capacitance $C = 20$ pF, and (loop) inductance $L = 50$ nH.
Let $f_{\max} = 10$ GHz be the maximum frequency of interest.
Find the minimum number n of lumped segments required for accurate modeling.

$L_{seg} = L/n$ and $C_{seg} = C/n$.

$$2\pi f_{\max} \le \frac{0.62}{\sqrt{L_{seg}C_{seg}}} = \frac{0.62n}{\sqrt{LC}} \Rightarrow n \ge \frac{2\pi f_{\max}\sqrt{LC}}{0.62} \text{ or } n \ge 10\frac{d}{\lambda_{\min}}$$

For the given numbers, $n_{\min} = 101$

Use of an insufficient number of segments leads to artificial filtering and phase distortion of the transmission-line response



The load response for a source and load match-terminated, lossless transmission line is:

$$V_L = 0.5\exp\left(-j\frac{2\pi\ell}{\lambda}\right)$$

If done properly, distributed RLCG circuit modeling of MTLs works

Such an approach is preferable when:

- **the wire resistance must be taken into account; (as already mentioned, some SPICE vendors provide lossy line modeling through extensions of the exact model mentioned earlier);**
- **the line is electrically short, and a few lumped-circuit segments are sufficient for its modeling;**
- **the MTL exhibits non-uniformity (variable cross section) along its axis;**
- **modeling of radiation coupling to the MTL is desired.**