Indian Institute of Science Education and Research, Trivandrum

PHY-321 **Stat-Mech problem set 3** **6.2.2015**

Note: Deadline: 16 February 2015 (12:30 PM)

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1. Using the microcanonical ensemble, compute the Helmholtz free energy $F(T, N)$ as a function of temperature for a system of N identical but distinguishable particles, each of which has two energy levels. Explore the limits $T \to 0$ and $T \to \infty$ of the energy, entropy and the occupation numbers.

2. Consider the system of N identical but distinguishable particles, each of which has two energy levels with energy 0 or $\epsilon > 0$. The uppper energy level has a $g-$fold degeneracy while the ground state is non-degenerate. [g-fold degenerate states mean that the g number of energy states have the same energy level.] The total energy of the system is U.

   (a) Using the microcanonical ensemble, find the occupation numbers n_+ and n_0 in terms of the system temperature. (n_+ corresponds to the uppper level and n_0 corresponds to the lower level.)

   (b) Consider the case $g = 2$. If the system energy $U = 0.75\,N\,\epsilon$ and is brought into contact with a bath at constant temperature $T = 500$ K, in which direction does the heat flow?

3. A system of N three-level particles has a Hamiltonian of the form:

$$H = -h \sum_{i=1}^{N} S_i \qquad S_i = 1, 0, -1 \tag{1}$$

   where h is positive constant. If n_S is the average number of particles in the state S ($S = 1, 0, -1$), use the microcanonical ensemble to find the ratio n_{-1}/n_1 in terms of the temperature in the limit of $N \to \infty$. Find the Helmholtz free energy.

4. A solid contains N mutually noninteracting nuclei of spin 1. Each nucleus can therefore be any of the three quantum states labeled by the quantum number m, where $m = 0, \pm 1$. Because of the electronic interactions with internal fields in the solid a nuceus in the state $m = 1$ or in the state $m = -1$ have the same energy $\epsilon > 0$, while its energy in the state $m = 0$ has zero energy.

   Derive the expression for the entropy of the N nuclei as a function of temperature T, and an expression for the heat capacity in the limit $\epsilon/(k_B T) \ll 1$.