Indian Institute of Science Education and Research, Trivandrum

PHY-321 **Stat-Mech problem set 6** **24.3.2015**

Note: Deadline:6 April 2015 (12:30 PM)

1. **Classical Hard sphere gas**

   Calculate b_2 and b_3 (Virial coefficients) for a classical hard sphere with hard-sphere diameter a.

   Express the equation of state of a classical hard-sphere gas in the form of a virial expansion. Include terms up to the third virial coefficient.

2. **Lennard Jones Potential**

   The general Lennard-Jones potential is

   $$V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}} \tag{1}$$

   Calculate b_2 and b_3 (Virial coefficients) for the above interaction potential.

3. Consider a quantum-mechanical gas of non-interacting spin zero bosons, each of mass m which are free to move within volume V.

   (a) Find the energy and heat capacity in the very low temperature. Discuss why it is appropriate at low-temperatures to put the chemical potential equal to zero.

   (b) Show how the calculation is modified for a photon gas.

4. In the class we derived the following relation for photon gas:

   $$PV = \frac{1}{3}U \tag{2}$$

   Using thermodynamic arguments, and the above equation, obtain the dependence of the energy density on the temperature for a photon gas.

5. A He-Ne laser generates a quasi-monochromatic beam at 6328 Angstrom. The beam has an output power of $1\,mW$, a divergence angle of 10^{-4} radians, and a spectral linewidth of 0.01 Angstrom. If a black body with an area of 1cm^2 were used to generate such a beam after proper filtering, what should its temperature be?

6. **n-dimensional universe**

   Calculate the average energy of black body radiation in an n-dimensional universe.

7. Graphite has a layered crystal structure in which the coupling between the carbon atoms in different layers is much weaker than that between the atoms in the same layer. Experimentally it is found that the specific heat is proportional to T (temperature) at low temperatures. How can the Debye theory be adopted for such an explanation?

8. Consider an ideal Fermi particles at temperature T.

   (a) Write the probability $p(n)$ that there are n particles in a given single particle state as a function of the mean occupation number $\langle n \rangle$.

   (b) Find the root-mean-square fluctuation in the occupation number of a single particle states as a function of the mean occupation number $\langle n \rangle$. Sketch the result.