Indian Institute of Science Education and Research, Trivandrum

PHY-321 **Stat-Mech quiz 1** **29.1.2015**

Points to note:

1. This is a closed book quiz. Each problem carries 5 points; total is 20 points.
2. Be clear; Be specific; Be neat.
3. Useful formulas are given at the end of the last question.

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1. Assuming that the entropy S and number of states Ω are related through an arbitrary functional form:

$$S = f(\Omega)$$

show that the additive nature of S and the multiplicative character of Ω necessarily require that the function $f(\Omega)$ be of the form

$$S = k_{_B} \ln \Omega$$

2. How much heat must be added to a system at $298\,K$ for the number of accesible states to increase by a factor of 10^6?

3. Suppose that Ω varies as

$$\Omega = A\, e^{\gamma (U\, V)^{1/2}}$$

where A, γ are constants. How does the temperature vary as a function of energy (U)? For what value of the energy is the temperature zero?

4. A system is composed of two harmonic oscillators each of natural frequency ω_0 and each having permissible energies $(n + 1/2)\hbar\omega_0$, where n is non-negative integer. The total energy of the system is $E = N\hbar\omega_0$, where N is a positive integer. How many microstates are available to the system? What is the entropy of the system?

   A second system is also composed of two harmonic oscillators each of natural frequency $2\omega_0$. The total energy of this system of $\tilde{E} = \tilde{N}\hbar\omega_0$ where $\tilde{N}$ is an even integer. How many microstates are available to the system? What is the entropy?

   What is the entropy of the system composed of the two preceeding sub-systems (no exchange of energy and particles)? Express the entropy as a function of E and $\tilde{E}$.

Useful formulae

1. Boltzmann constant $k_{_B} = 1.380650310^{-23} m^2\, kg\, s^{-2} K^{-1}$
2. Stirling's approximation $\ln n! \simeq n \ln n - n$
3. $\ln(10) = 2.30258509$
4. $\frac{1}{T} = \frac{\partial S}{\partial U}$