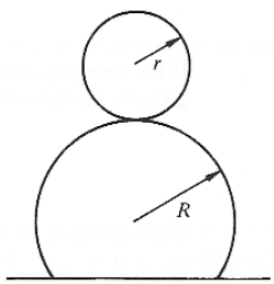
EP 222: Classical Mechanics
Tutorial Sheet 2

This tutorial sheet contains problems related to the calculus of variations, and Hamilton's principle.

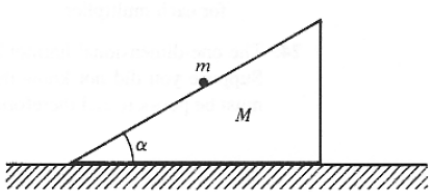
1. Show that the geodesics of a spherical surface are great circles, i.e., circles whose centers lie at the center of the sphere.

2. A uniform hoop of mass m and radius r rolls without slipping on a fixed cylinder of radius R. The only external force is that of gravity. If the smaller cylinder starts rolling from rest on top of the bigger cylinder, use the method of Lagrange multipliers to find the point at which the hoop falls off the cylinder.



3. A point mass is constrained to move on a massless hoop of radius a fixed in a vertical plane that rotates about its vertical symmetry axis with constant angular speed ω. Obtain the Lagrange equations of motion assuming the only external forces arise from gravity. What are the constants of motion? Show that if ω if greater than a critical value ω_0, there can be a solution in which the particle remains stationary on the hoop at a point other than at the bottom, but that if $\omega < \omega_0$, the only stationary point for the particle is at the bottom of the hoop. What is the value of ω_0?

4. A particle of mass m slides without friction on a wedge of angle α and mass M that can move without friction on a smooth horizontal surface, as shown in the figure. Treating the constraint of the particle on the wedge by the method of Lagrange multipliers, find the equation of motion for particle and wedge. Also obtain an expression for the forces of constraint. Calculate the work done in time t by the forces of constraint acting on the particle and on the wedge. What are the constants of motion for the system?



5. The one-dimensional harmonic oscillator has the Lagrangian $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$. Suppose you did not know the solution to the motion but realized that the motion must be periodic and therefore could be described by a Fourier series of the form

$$x(t) = \sum_{j=0} a_j \cos j\omega t,$$

(taking $t = 0$ at a turning point) where ω is the unknown angular frequency of the motion. This representation for $x(t)$ defines a many-parameter path for the system point in configuration space. Consider the action integral I for two points t_1 and t_2 separated by the period $T = \frac{2\pi}{\omega}$. Show that with this form for the system path, I is extremum for nonvanishing x only if $a_j = 0$, for all $j \neq 1$, and only if $\omega^2 = k/m$.