PH403: Quantum Mechanics I
Tutorial Sheet 1

This tutorial sheet contains problems on the application of the uncertainty principle, Bohr's model of hydrogen atom, and wave packets.

1. Compute the energy levels for a hydrogen atom by assuming that the electron moves in circular orbits around the nucleus such that the circumference of an orbit is an integral number of de Broglie wavelengths. Show that this condition also amounts to quantization of the angular momentum of the electron.

2. Using Heisenberg uncertainty principle, show that the classical picture embodied in the Bohr's model of the atom is incompatible with the uncertainty relation unless we consider orbits corresponding to very large principal quantum numbers n.

3. Consider a one-dimensional simple Harmonic oscillator of frequency ω and mass m. Using the uncertainty principle, show that the energy of its ground state will be $\frac{1}{2}\hbar\omega$. Note that this ground state energy is also called "zero-point" energy.

4. Suppose at time $t = 0$, a free particle wave packet is described by a Gaussian momentum distribution
$$g(k) = \frac{\sqrt{a}}{(2\pi)^{1/4}} \exp(-\frac{a^2}{4}(k - k_0)^2),$$
where a and k_0 are constants. Obtain the corresponding initial wave packet $\psi(x, t = 0)$. Is it localized? Does it satisfy the uncertainty principle? Describe the time evolution of this system by obtaining and analyzing $\psi(x, t)$. To do this problem, you will find the Gaussian integral $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ useful.