PH403: Quantum Mechanics I
Tutorial Sheet 3

1. Consider a two-dimensional isotropic simple harmonic oscillator with $V(x, y) = \frac{1}{2}m\omega^2(x^2 + y^2)$. Discuss the degeneracy associated with its various energy levels.

2. In a three-dimensional problem, consider a particle of mass m, and of potential energy

$$V(x, y, z) = \frac{m\omega^2}{2}\left[\left(1 + \frac{2\lambda}{3}\right)(x^2 + y^2) + (1 - \frac{4\lambda}{3})z^2\right]$$

   where $\omega (> 0)$ and λ $(0 \leq \lambda < \frac{3}{4})$ are constants.

   (a) Obtain the eigenvalues and the eigenvectors of the Hamiltonian.

   (b) Calculate and discuss, as functions of λ, the variation of the energy, the parity and the degree of degeneracy of the ground state and the first two excited states.

3. Consider the free-particle as a central force problem. Set up the time independent Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi,$$

   in spherical polar coordinate system, and examine the nature of the solutions. What will happen if we introduce a constant potential V_0 everywhere?

4. Consider a three-dimensional isotropic simple harmonic oscillator of mass m and potential $V(\mathbf{r}) = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2) = \frac{1}{2}m\omega^2 r^2$. Note that this system is also spherically symmetric. Set up the Schrödinger equation for the isotropic 3D oscillator in the spherical polar coordinates, and obtain the eigenvalues.

5. Consider a particle moving in a cylindrically symmetric potential $V(\rho, \phi, z) = V(\rho)$, where ρ, ϕ, and z are cylindrical polar coordinates. Show that the wave function for the system can be written in the form

$$\phi(\rho, \phi, z) = J(\rho)e^{il\phi}e^{ikz},$$

   where l is an integer, and $J(\rho)$ is a function of ρ. Obtain the differential equation satisfied by $J(\rho)$.