Chapter - 1

The Basics

9016036436

(I) Why Quantum Theory?

The most important question which one should ask is as to why quantum mechanics was needed at all? We had a pretty successful theory of material particles in form of ~~Newtoni~~ Newtonian mechanics, and the electromagnetic theory as embodied in Maxwell equations described phenomena ~~such~~ related to charged particles, electro-magnetic waves, and their interactions very well. So why bother ~~to~~ about a new theory called quantum theory?

The fact of the matter is that at the beginning of the twentieth century it was realized that our understanding of the physics at the microscopic level was far from complete. Some of the phenomena which were poorly understood were:

(contd.)

(i) blackbody spectrum

(ii) spectra of various atoms and molecules, quantization of their energy levels

(iii) photoelectric effect

~~and so on~~ and several other phenomena.

Early explanations of these phenomena were given by different people using different approaches, the common feature of which was the notion of quantum.

For example, the energy levels of the hydrogen atom were explained ~~using~~ by N. Bohr using his hypotheses which implied quantization of energy. Black body spectrum was explained by Max-Planck who assumed that energy was quantized. Similarly the important assumption involved in the Einstein's work on photoelectric effect was the quantization of energy. of light waves. Thus the notion of "quantum" emerged from various directions

Quantum Nature of Light :

Planck's explanation of blackbody radiation and Einstein's explanation of the photoelectric effect both relied on the quantization of light energy with smallest unit of ~~Energy~~ energy being

$$E_{min} = h\nu = \hbar\omega \tag{1}$$

where $h/\hbar$ are constants called the Planck's constant while ν/ω are the frequency/ angular frequency of ~~light~~ the radiation involved. ~~These~~ Thus it was postulated that these indivisible entities with energy $\hbar\omega$ were objects called photons and that e-m radiation was composed of these objects. It was further postulated that photons also carried momentum given by

$$\vec{p} = \hbar\vec{k} \tag{2}$$

where $\vec{k} \equiv$ wave number is given by

$$\vec{k} = \frac{2\pi}{\lambda}\hat{e} \tag{3}$$

$\hat{e} \equiv$ unit vector in the direction of propagation of light.

With momenta associated with e-m. radiation, it in fact has particle-like properties. Indeed, the particle-like properties of photons were verified ~~by~~ in the Compton effect experiment. Thus e-m. radiation exhibits both wave-like (interference, diffraction, ..) and particle-like (Blackbody radiation, Compton effect, photo-electric effect) properties. In other words it exhibits wave-particle duality.

De Broglie's Hypothesis:

Inspired by the success of the Bohr's model of the hydrogen atom, and the ideas of wave-particle duality of e-m. radiation propounded by Einstein and Max-Planck, Louis de Broglie argued that wave-particle duality is a fundamental property of all matter, and not just light. He argued that even material particles will exhibit wavelike properties with the wave length

$$\lambda = \frac{h}{p} \quad \text{—} \quad (4)$$

where p is the momentum of the particle. Note that this relation is equivalent to Eq.(2) defined for light: So de Broglie simply generalized the relations propounded by Einstein-Planck for light, to the domain of material particles. ~~He further~~ Louie de Broglie named the waves associated with the material particles "matter waves".

Heisenberg's Uncertainty Principle:

~~Another~~ Heisenberg proposed that at microscopic level there are unsurmountable uncertainties associated with physical measurements which are independent of the precision of the apparatus used. In other words these uncertainties effectively originate from a fundamental physical law which he called the "uncertainty principle". The original formulation of the uncertainty principle was:

(contd.)

"If one can measure the x component of the momentum with an uncertainty Δp, you cannot, at the same time, know its x-position more accurately than $\Delta x = \frac{h}{\Delta p}$" where h is Planck's constant. In other words

$$\Delta x \Delta p \sim h \quad — ⑤$$

In other words, if ~~we~~ one measures the position of an object to a great certainty, its momentum becomes uncertain, and vice versa. The same result applies to other components of momenta/position as well. We will learn that a more general formulation of the uncertainty principle involves any pair of canonically conjugate variables. We will also learn that the uncertainty principle and wave-particle duality are deeply related to each other.

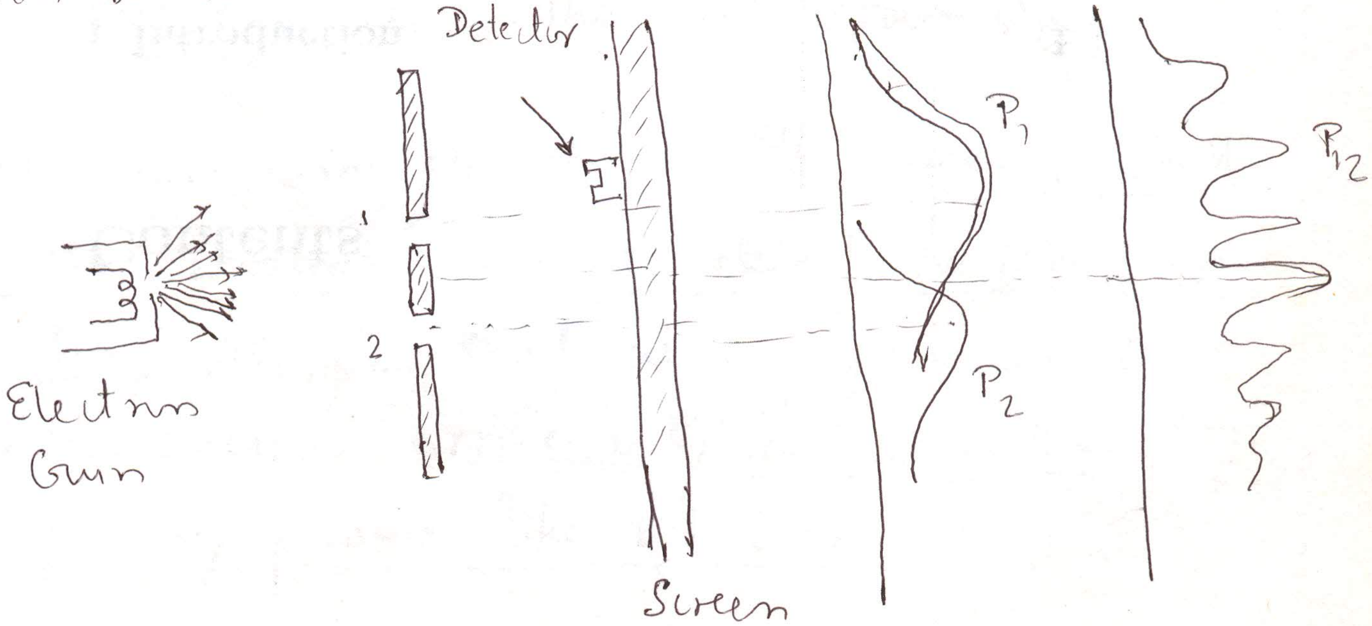
Some Quantum Experiments:

Here we will discuss some ~~expo~~ experiments which ~~[illegible]~~ illustrate the notions of wave-particle duality and the uncertainty principle. Then we will ~~illustrate~~ discuss the connections between the two pictures.

~~We will realise that~~

(I) Young's Double Slit Experiment with Electrons:

Let us consider the experimental set up given below



Electrons emitted from a source are made to pass through a double slit arrangement and then they impinge upon a screen on which a detector is placed which ~~can locate the position on the screen~~ is free to move along the screen. If a photographic plate is kept on the screen, then after sufficient exposure, it can be developed to observe the intensity pattern of the scattered electrons. One finds that, akin to the double slit experiment performed with the light, the intensity pattern exhibits dark and bright fringes. In other words, scattered electrons exhibit wavelike phenomenon of interference. This is astounding.

However, the detector on the screen, which can be ~~detected~~ used to detect even a single electron, reports that at any point on the screen electrons arrive only one at a time. This is a very particle-like behavior.

When the experiment above is done with a very low intensity beam of electrons so that ~~only~~ only one electron is emitted at a time, still after sufficient exposure one gets an interference pattern on the screen. Under those circumstances, the question is: what is interfering with what?

Recall that in the optical experiment beams coming from different slits interfere on the ~~sc~~ screen to give rise to fringes. Here we have material particles which arrive on the screen one-at-a-time, then how can there is interference?

Since electron is a material particle, it must be passing through either hole 1 or 2 before hitting the screen. So if we add the intensities of the electrons scattered from two holes we should get the result of the double slit experiment.

So we perform the experiment with holes 2/1 blocked to obtain the intensity patterns P_1/P_2. If the intensity pattern obtained with both the holes open is P_{12}, we note that

$$P_{12} \neq P_1 + P_2$$

Because P_1 & P_2 are essentially Gaussian distributions centered around holes 1/2 while P_{12} exhibits interference fringes.

Thus, when we know for sure which hole did the electron pass through, interference pattern gets wiped out.

We can try a more sophisticated experiment which will not involve blocking any hole, but will be able to track which hole a given electron went through. We can put a detector each close to both the holes which only track the electrons by say "shining light" on them but does not absorb them. Then by doing coincidence measurements with the detector on screen we can tell which electron passed through which hole. If the intensity pattern of electrons passing through hole 1/2 is P_1'/P_2', we find P_1'/P_2' similar to P_1/P_2. Moreover, if the total intensity is P_{12}', we find

$$P_{12}' = P_1' + P_2'$$

and no interference pattern.

Thus we conclude:

When the double slit experiment is performed ~~in a way~~ so that we do not know which hole the individual electrons have passed through, we obtain interference pattern.

However, whenever we repeat the experiment where we track the electrons through which hole they passed, the interference pattern is wiped out.

Recall that in optics interference pattern is obtained whenever we can add the amplitudes of the light wave, and is not seen whenever we add the intensities.

So if we define an 'amplitude' for each electrons journey fr the gun to the screen as ψ and further classify

$\psi_1 \equiv$ amplitude if the electron passed through hole 1

$\psi_2 \equiv$ amplitude if the electron passed through hole 2

Then when interference pattern is obtained

$$P_{12} \propto |\psi_1 + \psi_2|^2 \equiv \text{amplitudes are added}$$

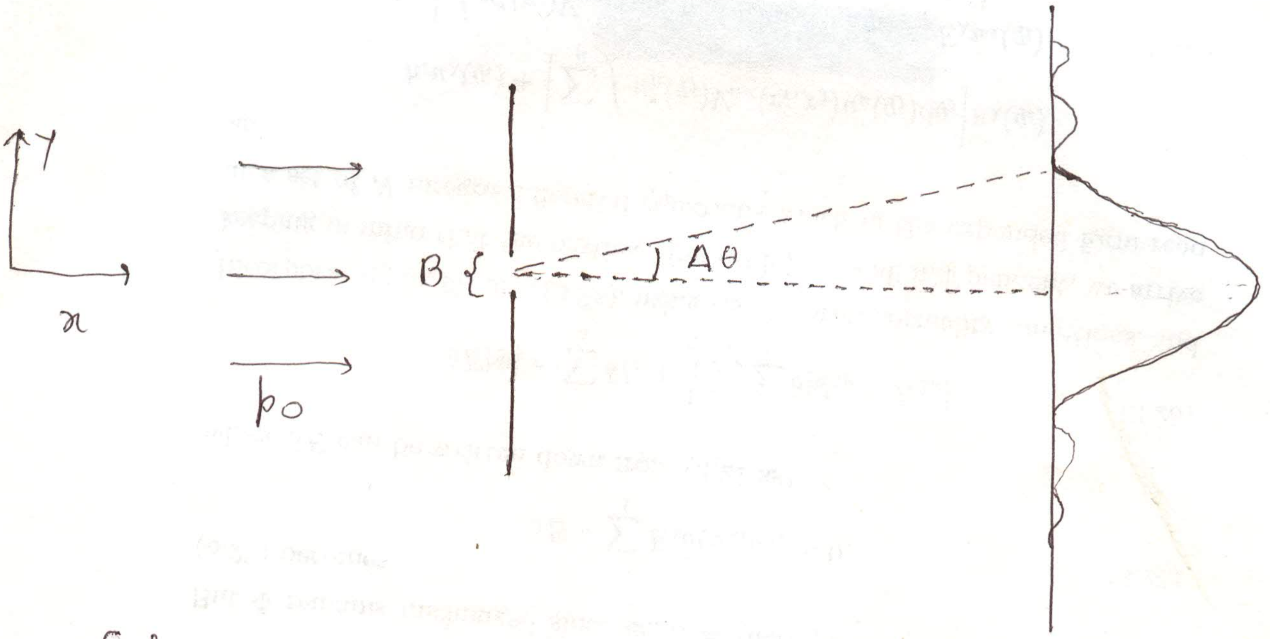
and when we know which hole electron passed through

$$P_{12}' \propto (|\psi_1|^2 + |\psi_2|^2) \equiv \text{intensities are added}$$

The deepest mystery of the nature is that how is this decision made by electrons as to what rule to follow?

Obviously, the act of measurement disturbs the system in such a way that the interference pattern is wiped out. This is consistent with the spirit of the uncertainty principle.

(II) Single-slit experiment and position momentum uncertainty:



Let us examine position-momentum uncertainty from ~~the pt~~ an experimental point of view. We envision a beam of electrons whose position we want ascertain at a given time. We arrange a single-slit experiment for the purpose, as shown in the figure. An electron beam is emitted from a gun, and we assume that it is traveling in the x-y plane. ~~B~~ To measure the position of the electrons, we place a single slit of width B, at a given point, with a screen a certain distance away from the slit. All the electrons which hit the screen must have passed through the slit, thereby determining their x coordinates precisely when they passed through the slit. But what about their y coordinates?

We further assume that the beam is well collimated so that the electron momenta are along the x-direction so that $p_y = p_z = 0$.

Now if the width of the slit is large the electrons will pass throught it undeflected implying that $p_y \approx 0$ while they pass through the slit. But the uncertainty in the y-coordinate of those electrons is huge and of the order of the slit width, i.e. $\Delta y \sim B$.

Now to measure the y-coordinate of those electrons, we must make the slit as narrow as possible. However, once we do that electrons will begin to diffract at the slit leading to the formation of a diffraction pattern at the screen. This means that now the y-component of their momentum p_y will become uncertain and will have an uncertainty, say, Δp_y. If the initial momentum of the electrons is p_0, the y ~~uncertaint~~ momentum of electrons which hit the screen at an angle $\Delta\theta$ will be $p_y \approx \cancel{p}\Delta p_y \simeq p_0\Delta\theta$. If $\Delta\theta$ corresponds to the position of the first minimum of the diffraction pattern then from the rules of diffraction

$$\Delta\theta = \frac{\lambda}{B}$$

so

$$\Delta p_y \approx p_0\frac{\lambda}{B} \quad — ⑥$$

Where λ is the wavelength of the electrons

But $\lambda = \cancel{p}\frac{h}{p_0}$ (as per de Broglie)

and $\Delta y = B$ (as argued earlier)

so we obtain from ⑥

$$\Delta p_y \approx \frac{h}{p_0}\frac{p_0}{\Delta y}$$

$$\Rightarrow \boxed{\Delta p_y \Delta y \approx h} \quad — ⑦$$

which is precisely the Heisenberg uncertainty principle. Thus we conclude that wave-particle duality of de Broglie and uncertainty principle of Heisenberg are fully consistent with each other.

~~Also~~ So far our discussion has been qualitative. Next question we must pose is as to what are the quantitative laws of quantum mechanics? What are the equations which govern the dynamics of nature at the microscopic level?