Time dependent perturbation

(contd.)

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$$H = H_0 + V(t)$$

$$H_0 |n\rangle = E_n |n\rangle$$

$$|\alpha\, t\rangle = \sum_n c_n(t)\, e^{-iE_n t/\hbar} |n\rangle$$

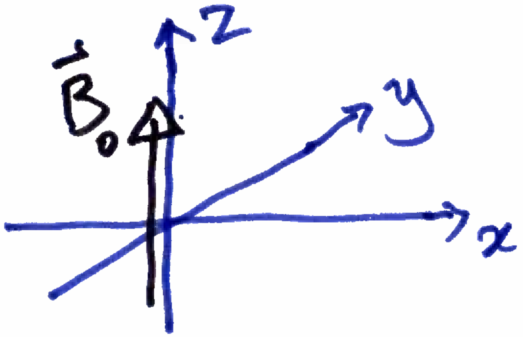
$$i\hbar \frac{d}{dt} c_n(t) = \sum_m V_{nm}\, e^{i\omega_{nm}t}\, c_m(t)$$

Two-level example $|c_2(t)|^2 = |A|^2 \sin^2(\Omega_+ - \Omega_-)t$

$\hookrightarrow A(\omega, \omega_{21})$

Spin magnetic resonance

$$\vec{B} = B_0 \hat{z} + B_1 (\hat{x} \cos\omega t + \hat{y} \sin\omega t)$$



$$H = -\vec{\mu} \cdot \vec{B}$$

$$\vec{\mu} = \frac{e}{mc} \vec{S} = \frac{e}{mc} \frac{\hbar}{2} \vec{\sigma}$$

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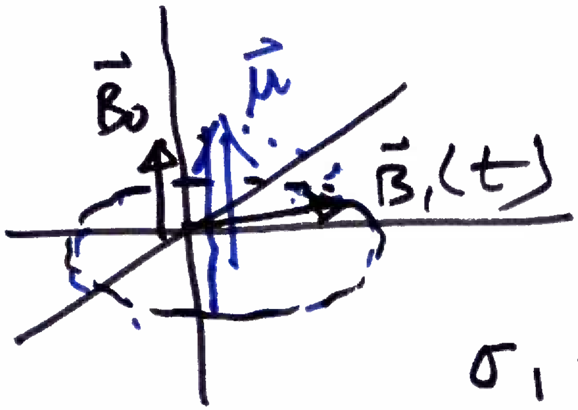
$$H_0 = -\mu_z B_0 = -\frac{e}{mc} B_0 \frac{\hbar}{2} \sigma^3$$

$$\sigma^3 |\pm\rangle = \pm |\pm\rangle$$

$$|E_+ - E_-| = \frac{e}{mc} B_0 \hbar$$

$$\left\{ \begin{array}{l} |+\rangle \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ |-\rangle \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{array} \right.$$

The time dependent part is a rotating magnetic field in x-y plane



$$V(t) = -\vec{\mu}\cdot\vec{B}_1$$

$$= B_1 \frac{e\hbar}{2mc}\left(\sigma_1 \cos\omega t + \sigma_2 \sin\omega t\right)$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \rightarrow \sigma_1 \equiv |+\rangle\langle-| + |-\rangle\langle+|$$

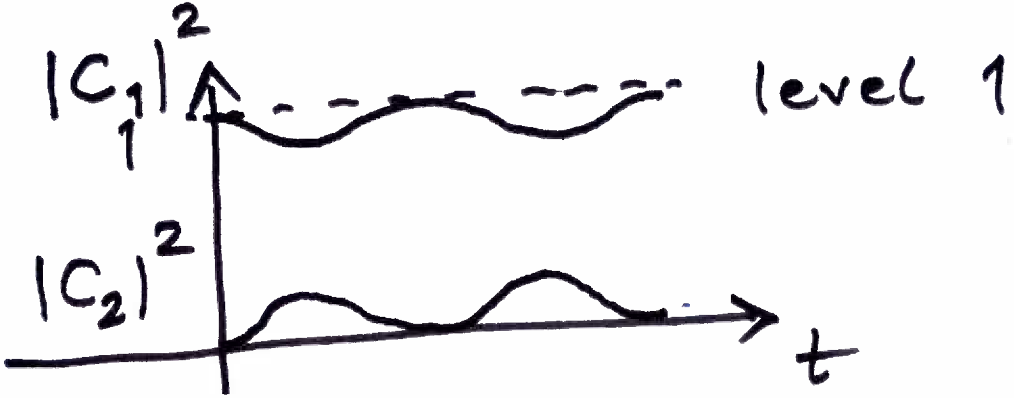
$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \rightarrow \sigma_2 \equiv -i|+\rangle\langle-| + i|-\rangle\langle+|$$

$$V(t) = -\left(\frac{e\hbar B_1}{2mc}\right)\Big\{\cos\omega t\left(\underline{|+\rangle\langle-|} + |-\rangle\langle+|\right) + (-i)\sin\omega t\left(\underline{|+\rangle\langle-|} - |-\rangle\langle+|\right)\Big\}$$

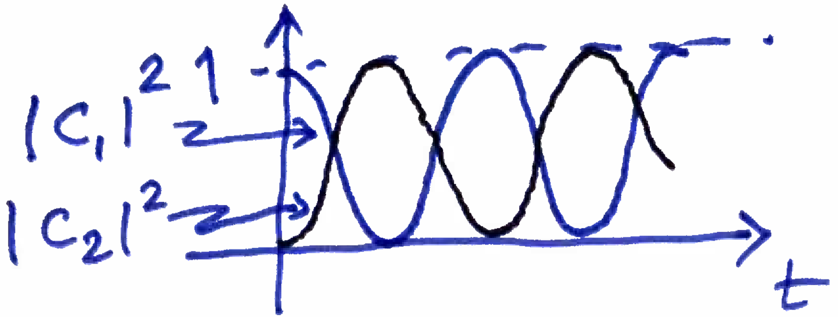
$$\rightarrow e^{-i\omega t}|+\rangle\langle-| + e^{i\omega t}|-\rangle\langle+|$$

Thus this is exactly like the toy problem with $\gamma \to \frac{e\hbar B_1}{2mc}$ $\quad \omega \to -\omega$

"ESR" I. I. Rabi 1930...



$\omega = \omega_{21} \equiv (E_2 - E_1)/\hbar$



Iterative solution :

$$i\hbar \frac{d}{dt} c_n(t) = \sum_m V_{nm} e^{i\omega_{nm}t} c_m(t)$$

Let at $t=0$ only $n=1$ be occupied

Perturbation assumption: V_{nm} are all "small"

$$V_{nm} << E_n$$

First assume R.H.S. above same as initial condition. We had $c_n^{(0)} = \delta_{n1}$

$$c_n^{(1)}(t) = -\frac{i}{\hbar}\int^t V_{n1} e^{i\omega_{n1}t'} dt'$$

It is easy to see that, using the general form,

$$c_n^{(2)}(t) = -\frac{i}{\hbar}\sum_m \int V_{nm}\, e^{i\omega_{nm}t'}\, c_m(t')dt'$$

$$\rightarrow -\frac{i}{\hbar}\sum_m \int V_{nm}\, e^{i\omega_{nm}t'}\, c_m^{(1)}(t')\,dt'$$

$$= \left(-\frac{i}{\hbar}\right)^2 \sum_m \int_0^t V_{nm}\, e^{i\omega_{nm}t'}\, dt' \int_0^{t'} V_{m1}\, e^{i\omega_{m1}t''}\, dt''$$

Thus we can compute

$$c_n(t) = c_n^{(0)}(t) + c_n^{(1)}(t) + c_n^{(2)}(t) \dots$$

where $c_n^{(k)} \sim |V|^k$

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Example: Apply iterative scheme to 2-level system

$$c_2^{(1)}(t) = -\frac{i}{\hbar}\int_0^t V_{21}\, e^{i\omega_{21}t'}\, dt'$$

$$= -\frac{i}{\hbar}\int_0^t \gamma\, e^{-i\omega t' + i\omega_{21}t'}\, dt'$$

... in the notation given earlier

$$= \left(-\frac{i}{\hbar}\right)\frac{(-i)\,\gamma}{\omega-\omega_{21}}\left(e^{i(\omega_{21}-\omega)t}-1\right)$$

$$|c_2^{(1)}(t)|^2 = \frac{\gamma^2}{\hbar^2}\frac{1}{(\omega-\omega_{21})^2}\, 4\sin^2\left(\frac{\omega_{21}-\omega}{2}\right)t$$

$$e^{ix}-1$$

$$e^{ix/2}\left(e^{ix/2}-e^{-ix/2}\right)$$

$$= e^{ix/2}\, 2i\sin\frac{x}{2}$$