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.... on Path Integral formulation

- No obvious advantage in ~~no~~ 1-particle QM
- Suited for transition amplitudes
  ... but can be adapted for a few static properties
- In QFT, useful tool for deriving <u>formulae</u>
  ... not so much specific numerican answers
- What can be solved?
  (i) kernel for H.O.
  (ii) Double slit experiment?

Bound state perturbation theory (contd.)

$$(\Delta E_n^{(\lambda)} - V)\,|\Delta n\rangle_\lambda = 0$$

Thus $\Delta E_n^{(1)} = \langle n^{(0)}|V|n^{(0)}\rangle$ to order λ

$$|\Delta n\rangle_n = K_n V |n^{(0)}\rangle \qquad (V - \Delta E_n)$$

where

$$K_n (E_n^{(0)} - H_0) \equiv \mathbb{1} - |n^{(0)}\rangle\langle n^{(0)}|$$

$$\Delta E_n^{(\lambda)} = \langle n^{(0)}|V|n\rangle$$

$$|\Delta n\rangle_\lambda = c_0 \cancel{|n^{(0)}\rangle} + K_n(\cancel{-\lambda}V + \Delta E_n)|n\rangle$$

$$(H_0 - E_n^{(0)})|\Delta n\rangle_\lambda = (\Delta E_n^{(\lambda)} \cancel{-\lambda}V)|n^{(0)}\rangle$$

$$(H_0+\lambda V)\left(|n^{(0)}\rangle+\lambda|\Delta n\rangle_\lambda\right)$$

$$=\left(E_n^{(0)}+\lambda\Delta E_n^{(\lambda)}\right)\left(|n^{(0)}\rangle+\lambda|\Delta n\rangle_\lambda\right)$$

$$\boxed{\begin{aligned}\Delta E_n^{(\lambda)} &= \langle n^{(0)}|V|n\rangle\\ |\Delta n\rangle_\lambda &= K_n(\lambda V-\Delta E_n^{(\lambda)})|n\rangle\end{aligned}}$$

Thus in first iteration,

$$\Delta E_n^{(\lambda)} = \langle n^{(0)}|V|n^{(0)}\rangle$$

and $|\Delta n\rangle_\lambda = K_n(\lambda V - \Delta E_n^{(\lambda)})|n^{(0)}\rangle$

Earlier we set up

$$E_n^{(\lambda)} = E_n^{(0)} + \lambda \Delta E_n^{(\lambda)}$$

Now expand $\Big\}$ $= E_n^{(0)} + \lambda \Delta E_n^{(1)} + \lambda^2 \Delta E_n^{(2)} \ldots$

Similarly

$$|n\rangle_\lambda = |n^{(0)}\rangle + \lambda |\Delta n\rangle_\lambda$$

Now expand

$$\lambda |\Delta n\rangle_\lambda = \lambda |n^{(1)}\rangle + \lambda^2 |n^{(2)}\rangle + \ldots$$

Then we can see that at a given order,

$$\Delta E_n^{(N)} = \langle n^{(0)} | V | \Delta n^{(N-1)} \rangle$$

$$|\lambda n^{(1)}\rangle = \cancel{k_n V | n^{(0)}\rangle} \Rightarrow k_n(\Delta E_n - V)|n^{(0)}\rangle$$

... Then we can go to next order for ΔE

$$\Delta E_n^{(2)} = \langle n^{(0)}| V |\lambda n^{(1)}\rangle$$

$$= \langle n^{(0)}| V k_n (\Delta E_n - V)| n^{(0)}\rangle$$

Thus we can derive

$$\boxed{\Delta E_n = \lambda \langle n^{(0)}|V|n^{(0)}\rangle + \lambda^2 \sum_{k\neq n} \frac{|V_{nk}|^2}{E_n^{(0)} - E_k^{(0)}}}$$

Detail:

$$\langle n^0|V \frac{1-\overset{|n^{(0)}\rangle\langle n^{(0)}|}{P}}{\underset{\sum_k |k^{(0)}\rangle\langle k^{(0)}|}{E_n^{(0)} - H_0}} (\lambda V - \Delta E_n)|n^{(0)}\rangle$$

$$\langle n^{(0)}|V|k^{(0)}\rangle\langle k^{(0)}|V|n^{(0)}\rangle$$

$$\Delta E_n \langle k^{(0)}| n^{(0)}\rangle \longrightarrow 0 \quad k \neq n$$

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Shift in vector to first order is

$$\boxed{|\Delta n^{(1)}\rangle = \sum_{k\neq n} \frac{\langle k^{(0)}|V|n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}} |k^{(0)}\rangle}$$

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